

Statistical tables

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Table A.1 Grubbs' test for a single outlier using mean and SD

Degrees of freedom	Significance level	
	5% (0.05)	1% (0.01)
2	1.15	1.15
3	1.48	1.50
4	1.71	1.76
5	1.89	1.97
6	2.02	2.14
7	2.13	2.27
8	2.21	2.39
9	2.29	2.48
10	2.36	2.56
11	2.41	2.64
12	2.46	2.70
13	2.51	2.76
14	2.55	2.81
15	2.59	2.85
16	2.62	2.89
17	2.65	2.93
18	2.68	2.97
19	2.71	3.00
20	2.73	3.03
30	2.92	3.26
40	3.05	3.39
50	3.14	3.49
100	3.38	3.75

$$\text{Test value} = \frac{|x - \bar{x}|}{s}$$

where x is the most extreme observation from the mean;

$\bar{x}$ is the mean of all observations including the possible outlier;

s is the SD of all observations including the possible outlier.

Table A.2 The t-table

Degrees of freedom	Significance level					
	10% (0.1)	5% (0.05)	2% (0.02)	1% (0.01)	0.2% (0.002)	0.1% (0.001)
1	6.31	12.71	31.82	63.66	318.31	636.62
2	2.92	4.30	6.97	9.92	22.33	31.60
3	2.35	3.18	4.54	5.84	10.21	12.92
4	2.13	2.78	3.75	4.60	7.17	8.61
5	2.02	2.57	3.37	4.03	5.89	6.87
6	1.94	2.45	3.14	3.71	5.21	5.96
7	1.89	2.36	3.00	3.50	4.79	5.41
8	1.86	2.31	2.90	3.36	4.50	5.04
9	1.83	2.26	2.82	3.25	4.30	4.78
10	1.81	2.23	2.76	3.17	4.14	4.59
11	1.80	2.20	2.72	3.11	4.03	4.44
12	1.78	2.18	2.68	3.06	3.93	4.32
13	1.77	2.16	2.65	3.01	3.85	4.22
14	1.76	2.15	2.62	2.98	3.79	4.14
15	1.75	2.13	2.60	2.95	3.73	4.07
16	1.75	2.12	2.58	2.92	3.69	4.02
17	1.74	2.11	2.57	2.90	3.65	3.97
18	1.73	2.10	2.55	2.88	3.61	3.92
19	1.73	2.09	2.54	2.86	3.58	3.88
20	1.72	2.08	2.53	2.85	3.55	3.85
25	1.71	2.06	2.49	2.78	3.45	3.72
30	1.70	2.04	2.46	2.75	3.39	3.65
40	1.68	2.02	2.42	2.70	3.31	3.55
60	1.67	2.00	2.39	2.66	3.23	3.46
120	1.66	1.98	2.36	2.62	3.16	3.37
Infinity	1.64	1.96	2.33	2.58	3.09	3.29
Confidence level	90%	95%	98%	99%	99.8	99.9%

Note: The quoted significance levels are for a two-sided test. To carry out a one-sided test, halve the significance level given at the top of the table.

Table A.3 Outlier test for two extreme observations, one high and one low

Degrees of freedom	Significance level	
	5% (0.05)	1% (0.01)
4	6.74	15.81
5	3.69	6.55
6	2.92	4.49
7	2.44	3.45
8	2.17	2.84
9	2.00	2.52
10	1.86	2.30
12	1.68	1.99
14	1.57	1.80
16	1.50	1.69
18	1.44	1.61
20	1.39	1.55
30	1.27	1.34
40	1.20	1.27
50	1.17	1.22
100	1.09	1.11

$$\text{Test value} = \frac{\text{SD for all observations}}{\text{SD excluding highest and lowest observations}}$$

Table A.4 Test for two extreme observations, both high or both low

Degrees of freedom	Significance level	
	5% (0.05)	1% (0.01)
4	2.10	2.16
5	2.41	2.50
6	2.66	2.79
7	2.87	3.02
8	3.04	3.22
9	3.18	3.40
10	3.31	3.56
12	3.56	3.82
14	3.75	4.02
16	3.90	4.18
18	4.04	4.33
20	4.17	4.47
30	4.60	4.97
40	4.87	5.32
50	5.08	5.53
100	5.63	6.07

Test value = $\frac{|\text{sum of two highest (or lowest) deviations}|}{s}$

where s is the standard deviation of all observations including the possible outliers.

Table A.5 Outlier test for residuals in simple and multiple regression

Sample size	5% (0.05) significance level									
	No. of coefficients in equation									
	1	2	3	4	5	6	8	10	15	25
5	1.92	1.74								
6	2.07	1.93								
7	2.19	2.08	1.94							
8	2.28	2.20	2.10	1.94						
9	2.35	2.29	2.21	2.10	1.95					
10	2.42	2.37	2.31	2.22	2.11	1.95				
12	2.52	2.49	2.45	2.39	2.33	2.24	1.96			
14	2.61	2.58	2.55	2.51	2.47	2.41	2.25	1.96		
16	2.68	2.66	2.63	2.60	2.57	2.53	2.43	2.26		
18	2.73	2.72	2.70	2.68	2.65	2.62	2.55	2.44		
20	2.78	2.77	2.76	2.74	2.72	2.70	2.64	2.57	2.15	
25	2.89	2.88	2.87	2.86	2.84	2.83	2.80	2.76	2.60	
30	2.96	2.96	2.95	2.94	2.93	2.93	2.90	2.88	2.79	2.17
35	3.03	3.02	3.02	3.01	3.00	3.00	2.93	2.97	2.91	2.64
40	3.08	3.08	3.07	3.07	3.06	3.06	3.05	3.03	3.00	2.84
45	3.13	3.12	3.12	3.12	3.11	3.11	3.10	3.09	3.06	2.96
50	3.17	3.16	3.16	3.16	3.15	3.15	3.14	3.14	3.11	3.04
60	3.23	3.23	3.23	3.23	3.22	3.22	3.22	3.21	3.20	3.15
70	3.29	3.29	3.28	3.28	3.28	3.28	3.27	3.27	3.26	3.23
80	3.33	3.33	3.33	3.33	3.33	3.33	3.33	3.32	3.31	3.29
90	3.37	3.37	3.37	3.38	3.38	3.38	3.36	3.37	3.36	3.34
100	3.41	3.41	3.40	3.40	3.40	3.40	3.40	3.40	3.39	3.38

↑
Simple regression
 $y = a + bx$

For simple regression :

$$\text{Test value} = \frac{\text{Maximum|Residual|}}{\text{RSD} \sqrt{1 - \frac{1}{n} - \frac{(X - \bar{X})^2}{(n-1)(\text{SD of } x)^2}}}$$

Table A.6 The F-test (two-sided)

		5% (0.05) significance level														
		Degrees of freedom for larger SD														
Degrees of freedom for smaller SD		1	2	3	4	5	6	7	8	9	10	12	15	20	60	Infinity
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6	976.7	984.9	993.1	1010	1018	
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	39.45	39.48	39.50	
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17	13.99	13.90	
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.36	8.26	
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.12	6.02	
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	4.96	4.85	
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.25	4.14	
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.78	3.67	
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.45	3.33	
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.20	3.08	
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	2.85	2.72	
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.52	2.40	
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.22	2.09	
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.67	1.48	
Infinity	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.39	1.00	

Table A.6 (Continued)

		1% (0.01) significance level														
		Degrees of freedom for larger SD														
Degrees of freedom for smaller SD		1	2	3	4	5	6	7	8	9	10	12	15	20	60	Infinity
1	16211	20000	21615	22500	23056	23437	23715	23925	24091	24224	24426	24630	24836	25253	25465	
2	198.5	199.0	199.2	199.2	199.3	199.3	199.4	199.4	199.4	199.4	199.4	199.4	199.4	199.4	199.5	199.5
3	55.55	49.80	47.47	46.19	45.39	44.84	44.43	44.13	43.88	43.69	43.29	43.08	42.78	42.15	41.83	
4	31.33	26.28	24.26	23.15	22.46	21.97	21.62	21.35	21.14	20.97	20.70	20.04	20.17	19.61	19.32	
5	22.78	18.31	16.53	15.56	14.94	14.51	14.20	13.96	13.77	13.62	13.38	13.15	12.90	12.40	12.14	
6	18.63	14.54	12.92	12.03	11.46	11.07	10.79	10.57	10.39	10.25	10.03	9.81	9.59	9.12	8.88	
7	16.24	12.40	10.88	10.05	9.52	9.16	8.89	8.68	8.51	8.38	8.18	7.97	7.75	7.31	7.08	
8	14.69	11.04	9.60	8.81	8.30	7.95	7.69	7.50	7.34	7.21	7.01	6.81	6.61	6.18	5.95	
9	13.61	10.11	8.72	7.96	7.47	7.13	6.88	6.69	6.54	6.42	6.23	6.03	5.83	5.41	5.19	
10	12.83	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97	5.85	5.66	5.47	5.27	4.86	4.64	
12	11.75	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20	5.09	4.91	4.72	4.53	4.12	3.90	
15	10.80	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54	4.42	4.25	4.07	3.88	3.48	3.26	
20	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96	3.85	3.68	3.50	3.32	2.92	2.69	
60	8.49	5.79	4.73	4.14	3.76	3.49	3.29	3.13	3.01	2.90	2.74	2.57	2.39	1.96	1.69	
Infinity	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62	2.52	2.36	2.19	2.00	1.53	1.00	

Test value = $\frac{\text{Larger } s^2}{\text{Smaller } s^2}$

Table A.6 The F-test (one-sided) for use in analysis of variance

Degrees of freedom for residual mean square		5% (0.05) significance level														
		Degrees of freedom for treatment mean square														
	1	2	3	4	5	6	7	8	9	10	12	15	20	60	Infinity	
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	246.0	248.0	252.2	254.3	
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.48	19.50	
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.57	8.53	
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.69	5.63	
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.43	4.36	
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.74	3.67	
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.30	3.23	
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.01	2.93	
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.79	2.71	
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.62	2.54	
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.38	2.30	
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.16	2.07	
20	4.35	3.49	3.10	2.87	2.71	2.60	2.49	2.45	2.39	2.35	2.28	2.20	2.12	1.95	1.84	
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.53	1.39	
Infinity	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.32	1.00	

Table A.6 (Continued)

1% (0.01) significance level															
Degrees of freedom for residual mean square		Degrees of freedom for treatment mean square													
		1	2	3	4	5	6	7	8	9	10	12	15	20	60
1	4052	5000	5403	5625	5764	5859	5928	5982	6022	6056	6106	6157	6209	6313	6366
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.45	99.48	99.50
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.87	26.69	26.32	26.13
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.20	14.02	13.65	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55	9.20	9.02
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.06	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	5.82	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.03	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.48	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.08	3.91
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.54	3.36
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.05	2.87
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.61	2.42
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	1.84	1.60
Infinity	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.47	1.00

Test value = $\frac{\text{Effect } MS}{\text{Residual } MS}$

Table A.7 Cochran's test for a high outlier amongst standard deviations

Number of observations per group		2		3		4		5		6	
Number of groups	Sig. level	5% (0.05)	1% (0.01)	5% (0.05)	1% (0.01)	5% (0.05)	1% (0.01)	5% (0.05)	1% (0.01)	5% (0.05)	1% (0.01)
2		—	—	0.98	1.00	0.94	0.98	0.91	0.96	0.88	0.94
3		0.97	0.99	0.87	0.94	0.80	0.88	0.75	0.83	0.71	0.79
4		0.91	0.97	0.77	0.86	0.68	0.78	0.63	0.72	0.59	0.68
5		0.84	0.93	0.68	0.79	0.60	0.70	0.54	0.63	0.51	0.59
6		0.78	0.88	0.62	0.72	0.53	0.63	0.48	0.56	0.45	0.52
7		0.73	0.84	0.56	0.66	0.48	0.57	0.43	0.51	0.40	0.47
8		0.68	0.79	0.52	0.62	0.44	0.52	0.39	0.46	0.36	0.42
9		0.64	0.75	0.48	0.57	0.40	0.48	0.36	0.43	0.33	0.39
10		0.60	0.72	0.45	0.54	0.37	0.45	0.33	0.39	0.30	0.36
11		0.57	0.68	0.42	0.50	0.35	0.42	0.31	0.37	0.28	0.33
12		0.54	0.65	0.39	0.48	0.33	0.39	0.29	0.34	0.26	0.31
13		0.52	0.62	0.37	0.45	0.31	0.37	0.27	0.32	0.25	0.29
14		0.49	0.60	0.35	0.43	0.29	0.35	0.26	0.30	0.23	0.27
15		0.47	0.58	0.34	0.41	0.28	0.33	0.24	0.29	0.22	0.26
16		0.45	0.55	0.32	0.39	0.26	0.32	0.23	0.27	0.21	0.25
17		0.43	0.53	0.31	0.37	0.25	0.30	0.22	0.26	0.20	0.23
18		0.42	0.51	0.29	0.36	0.24	0.29	0.21	0.25	0.19	0.22
19		0.40	0.50	0.28	0.34	0.23	0.28	0.20	0.24	0.18	0.21
20		0.39	0.48	0.27	0.33	0.22	0.27	0.19	0.23	0.17	0.21
25		0.33	0.41	0.25	0.31	0.19	0.22	0.16	0.19	0.14	0.17
30		0.29	0.36	0.20	0.24	0.16	0.19	0.14	0.16	0.12	0.15
35		0.26	0.33	0.18	0.21	0.14	0.17	0.12	0.14	0.11	0.13
40		0.24	0.29	0.16	0.19	0.13	0.15	0.11	0.13	0.10	0.11

Test value = $\frac{\text{largest } s^2}{\sum s^2}$

Table A.8 Analysis matrices for 2^2 , 2^3 and 2^4 factorial experiments

Trial	A	B	AB	C	AC	BC	ABC	D	AD	BD	ABD	CD	ACD	BCD	ABCD
1	−	−	+	−	+	+	−	−	+	+	−	+	−	−	+
2	+	−	−	−	−	+	+	−	−	+	+	+	+	−	−
3	−	+	−	−	+	−	+	−	+	−	+	+	−	+	−
4	+	+	+	−	−	−	−	−	−	−	−	+	+	+	+
5	−	−	+	+	−	−	+	−	+	+	−	−	+	+	−
6	+	−	−	+	+	−	−	−	−	+	+	−	−	+	+
7	−	+	−	+	−	+	−	−	+	−	+	−	+	−	+
8	+	+	+	+	+	+	+	−	−	−	−	−	−	−	−
9	−	−	+	−	+	+	−	+	−	−	+	−	+	+	−
10	+	−	−	−	−	+	+	+	+	−	−	−	−	+	+
11	−	+	−	−	+	−	+	+	−	+	−	−	+	−	+
12	+	+	+	−	−	−	−	+	+	+	+	−	−	−	−
13	−	−	+	+	−	−	+	+	−	−	+	+	−	−	+
14	+	−	−	+	+	−	−	+	+	−	−	+	+	−	−
15	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−
16	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

Table A.9 Rank order statistics for half-normal plots

8 Trials		12 Trials		16 Trials	
Order	Value	Order	Value	Order	Value
1	0.1597	1	0.1054	1	0.0787
2	0.3260	2	0.2128	2	0.1583
3	0.5042	3	0.3234	3	0.2393
4	0.7021	4	0.4388	4	0.3221
5	0.9344	5	0.5609	5	0.4075
6	1.2349	6	0.6926	6	0.4962
7	1.7239	7	0.8381	7	0.5893
		8	1.0047	8	0.6879
		9	1.2057	9	0.7941
		10	1.4730	10	0.9102
		11	1.9215	11	1.0404
				12	1.1914
				13	1.3763
				14	1.6256
				15	2.0509