Exercise 102

Determinantion of Young's modulus by beam bending

Requirements:

1. The elastic properties of the solid material. Hooke's law.
2. Beam bending
3. The least square method

Purpose

The puropse of the experiment is to observe the effects of the external forces on the solid material and to determine Young's modulus of certain materials.

Theoretical background

For sufficiently small elastic deformations (within the proportionality limit) Hooke's law is fulfilled. In the scalar form it is described by the following formula:

$$\sigma = E\varepsilon \tag{1}$$

where σ is the stress ($\sigma = F/S$, F is the force, perpendicular to the cross-sectional area S), E is the modulus of elasticity (Young's modulus, for example), ε is the strain.

One of the processes, described by Hooke's law, is beam bending. In our case the beam has rectangular cross-section with depth b and width d. Under external force $\boldsymbol{P}$ applied at the centre of the beam, the beam bends (see fig. 1). The centre of the beam lowers by distance λ (deflection) described by the following formula:

$$\lambda = \frac{Pl^3}{4Edb^3} \tag{2}$$

where l is the span (see fig. 1). It is assumed that $l \gg b$ and $l \gg d$.

The bending causes changes of lengths of the initially horizontal layers of the beam. The central layer of a symmetric beam is the neutral layer, i.e. it does not change its length—only its shape. All the layers below the neutral layer are in a state of tension, while the layers above side are compressed.

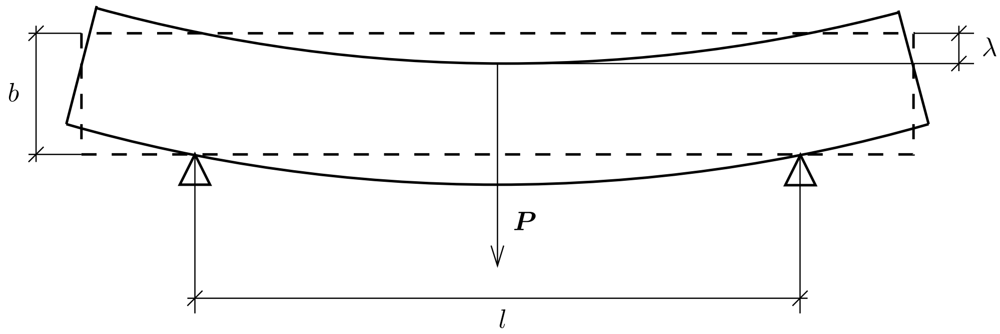

Figure 1: The beam and some quantities used in the calculations

In the experiment, deflections λ_i are measured for increasing forces P_i. The weight of the beam can be neglected. The relation between the deflection and the force should be linear:

$$\lambda = aP \tag{3}$$

Slope a can be found using the least square method. Using formulae (2) and (3) one can calculate Young's modulus:

$$E = \frac{l^3}{4db^3}\frac{P}{\lambda} = \frac{l^3}{4db^3}\frac{1}{a} \tag{4}$$

Experimental procedure

At the centre of the beam, placed on two supports several forces P_i are applied and the corresponding deflections λ_i are measured. Forces P_i are obtained by putting increasing the number of weights, which are supported by the beam. The deflections are measured by a dial meter. The dial meter should be zeroed before the first weight is put, by turning the outer frame. Results of all the measurements should be written together with their uncertainties.

1. Measure the span l with precision $\pm 1\,\mathrm{mm}$
2. Measure the beam's width d with precision $\pm 0.1\,\mathrm{mm}$
3. Measure the depth b in 10 evenly distributed places (why?)
4. Put the beam on the supports
5. Place the dial meter so that its gauge head could move up and down
6. Weigh one weight. One can assume the they are all identical.
7. **Carefully** put one weight on the special construction. Read the deflection λ_1.
8. Add the successive weights and measure the deflections
9. Repeat the procedure for all the beams

Analyse of the results

The report should contain:

1. A short description of the experiment, where the symbols used in the report are explained.
2. Calculation of the average value of the depth. Its uncertainty should be calculated using Student's t-distribution with 95% confidence interval.
3. Calculation, using the least square method, of factor a from equation (3)
4. Graph of function $\lambda(P)$
5. Calculation of Young's moduli (equation (4)) and their uncertainties:

$$\Delta E = E\left(\frac{3\Delta l}{l} + \frac{3\Delta b}{\bar{b}} + \frac{\Delta d}{d} + \frac{\Delta a}{a}\right) \tag{5}$$

6. Discussion of the results and the validity of the assumed approximations.