Exercise 131

Measurement of viscosity coefficient of a liquid based on the Poiseuille law

By: T. Majchrzak, M. Buczkowska

1. Theory

1. Viscosity. Newton’s law of internal friction. Flow of viscous liquid. [1], [2]
2. Flow in tubes with small diameters. Poiseuille law. [1], [2]
3. Principle of operation of the Mariotte bottle. [2]

2. Devices

1. Mariotte bottle with distilled water
2. Measuring beaker and spare beaker
3. Stopwatch
4. Calliper

3. Method

The flow of a liquid along tubes with small diameters (called capillaries or capillary tubes) is laminar (*why?*). The outflow of the liquid from a vessel through the capillary tube placed in the bottom part of the vessel can serve as an example. If the height of the liquid column i.e. if the hydrostatic pressure inducing the flow is constant, then the velocity of the liquid is also constant. The volume of the outflowing liquid V as well as its mass m is proportional to the time of the outflow t. (*What else influence this volume?*).

The constant height of the liquid column is ensured by means of so called Marriotte bottle shown in fig. 1. The value of h is equal to the distance between the level of the lower end of the vertical tube R and the level of the end of the capillary tube W. The viscosity coefficient η can be calculated from the following formula:

$$\eta = \frac{\pi r^4 \rho^2 hgt}{8l\left(m_{zc} - m_z\right)}$$

which can be derived from the Poiseuille law, where r - inner radius of the capillary, l – capillary length, ρ - density of the liquid, h – height of the liquid column, t – time of the outflow, m_{zc} – mass of the beaker with the liquid, m_z – mass of empty beaker, g – gravitational acceleration.

4. Course of the measurement

The experimental setup is shown schematically in figure 1. The Marriotte bottle contains distilled water. The water outflows through the capillary tube W which has small diameter 2*r*. The water is gathered in the beaker. Its mass, m_z, can be determined by use of analytical balance. Weighing the beaker with water when the measurement is finished which gives the mass m_{zc}, one can determine the mass of water which has flowed out during the time *t*. The time *t* is measured by means of the stopwatch. The height *h* as well as the length *l* is measured by use of millimetre ruler. The water temperature, which is equal to the room temperature, is read from a thermometer hanging in the classroom.

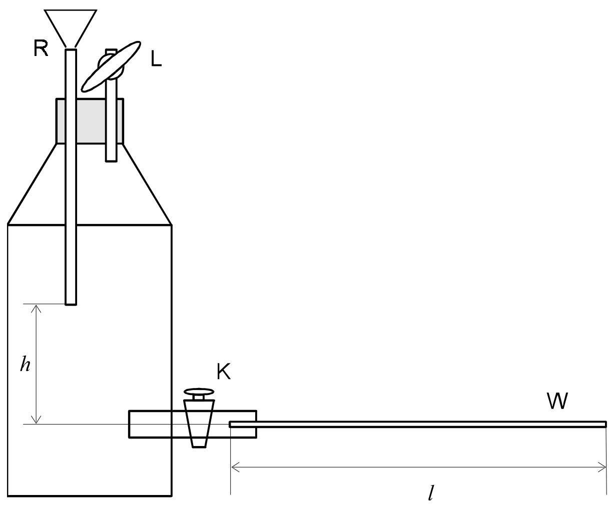

Fig. 1. Scheme of the experimental setup for the measurement of the viscosity coefficient of water

The density of water ρ at room temperature and the gravitational acceleration *g* can be read from tables of physical quantities. The capillary radius *r* and the uncertainty of its determination Δr are given on the label placed at the experimental setup. The tube L with the faucet serves for the filling of the Marriot bottle with water when the water level drops below its lower end R.
Attention! During the measurement the faucet L should be closed.

5. Experimental procedure

1. Check whether the water level in the Marriotte bottle is above the lower end of the vertical tube R .
2. Determine the beaker mass m_z on the analytical balance.
3. Open the faucet K at the beginning of the capillary tube and wait until the air bubbles appear at the lower end of the vertical tube R. The water outflowing during this time gather in spare glass.
4. Place the measuring beaker under the exit of the capillary tube W and switch on the stopwatch in order to measure the time t during which the beaker will fill with water up to c. $^2/_3$ of its height.
5. Estimate maximum absolute uncertainty Δt of the time t.
6. Determine the mass of the beaker with water m_{zc} and estimate absolute uncertainty Δm of the measurements of masses m_z and m_{zc}.
7. Measure the height h of the water column and the length l of the capillary and estimate the estimate absolute uncertainties Δh and Δl.
8. Read the ambient temperature T and the corresponding density of water ρ.
9. The results of measurements write in the table.

m_z [kg]	m_{zc} [kg]	Δm [kg]	t [s]	Δt [s]	r [m]	Δr [m]	h [m]	Δh [m]	l [m]	Δl [m]	T [K]	ρ [kg/m^3]

6. Preparation of the report

The report should contain:

1. Short description of the method applied, with necessary formulae and symbols.
2. Table with the measured values.
3. Calculation of the viscosity coefficient η .
4. Calculation of experimental uncertainty $\Delta\eta$ by use of the formula

$$\Delta\eta = \eta\left(\frac{4\Delta r}{r} + \frac{\Delta h}{h} + \frac{\Delta t}{t} + \frac{\Delta l}{l} + \frac{2\Delta m}{m_{zc} - m_z}\right).$$

5. Final result written in the form

$$\eta = \eta_{calc} \pm \Delta\eta ,$$

where η_{calc} is the calculated viscosity value.

6. Discussion of the obtained results and their comparison with the data contained in tables of physical quantities. Analysis of possible reasons of errors made during the experiment.

7. References

[1] B. Jaworski, A. Dietłaf, L. Miłkowska, *Kurs fizyki*, t.1, PWN, Warszawa, 1984
[2] J. Karniewicz, T. Sokołowski, *Podstawy fizyki laboratoryjnej*, skrypt PŁ, Łódź, 1996.