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Determination of the c_p/c_v ratio for air by Clement-Desormes method

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1. Theory

1. Ideal gas. Thermodynamic processes. Equation of state. Internal energy. Degrees of freedom. First law of thermodynamics and its application to gas processes. Heat capacity, specific heat [1,2].
2. Equipartition of energy. Average kinetic energy of molecules with respect to their internal structure. Formula for κ in different gases [1,2,3,4]
3. Clement-Desormes method of determining the c_p/c_V factor. Description of the thermodynamic processes involved. Derivation of c_p/c_V formula [3,4].

2. Equipment:

Measurement setup shown in the figure, pump, timer.

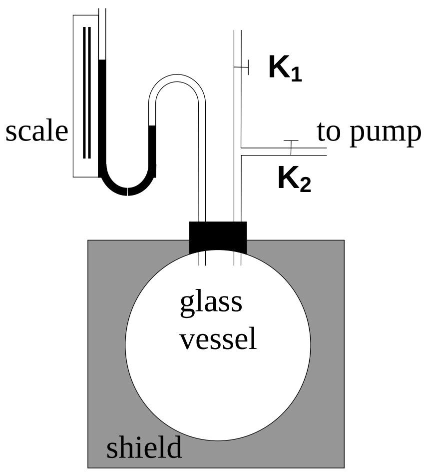

Fig. 1 : Measurement setup.

3. Measurement principle

The aim of the experiment is to determine the adiabatic coefficient $\kappa = c_p / c_V$, where c_p means the specific heat at constant pressure (isobaric process) and c_V is the specific heat at constant volume (isochoric process). The ratio $\kappa = c_p / c_V$ is also the exponent in the Poisson's formula $pV^{\kappa} = const$ which describes the adiabatic process. The value of $\kappa = c_p / c_V$ can be obtained utilizing changes in the parameters of a gas contained in a vessel with thermally insulated walls and undergoing particular thermodynamic processes (Clement-Desormes method).

When air at room temperature T_1 and pressure p_1 slightly larger than atmospheric pressure, contained in a vessel of volume V_1 (state I), is decompressed adiabatically to atmospheric pressure p_a, its temperature will decrease to $T_2 < T_1$ while the volume increases to V_2 (state II). This process is decribed by Poisson's formula $pV^{\kappa} = const$. After closing the vessel, the gas temperature returns to the value T_1 within several minutes and the pressure reaches the value p_2 (state III) at unchanged volume V_2. Since states I and III have the same temperature T_1, they belong to the same isotherm described by the formula $pV=const$. Both equations (of the adiabat and the isotherm) can be logarithmed and after that the complete derivative can be calculated. By replacing dp and dV with definite increments and eliminating the volume from the equations, the sought value of κ can be expressed by the pressure differences $\Delta p_1=p_1 - p_a$ and $\Delta p_2=p_2 - p_a$:

$$\kappa = \frac{\Delta p_1}{\Delta p_1 - \Delta p_2} . \tag{1}$$

The difference between the pressure in the vessel and the atmospheric pressure is proportional to the fluid height difference in an open fluid manometer (so-called U-tube): $\Delta p = \rho g h$, where ρ – fluid density, g – gravity constant, h – fluid height difference. It means that pressure differences Δp_1 and Δp_2 are proportional to h_1 and h_2 respectively and the expression (1) can be transformed to:

$$\kappa = \frac{h_1}{h_1 - h_2} . \tag{2}$$

4. Description of the experiment

The experimental setup consists of a glass vessel embedded in a thermal shield and closed with two outlets. A pipe with valves K_1 and K_2 is mounted in one of the outlets and the second outlet connects the vessel with a fluid manometer (Fig. 1). With closed K_1 valve and open K_2 valve, the pressure in the vessel is increased using a pump and the K_2 valve is then closed. After several minutes needed to equalize the gas temperature in the vessel with the ambient temperature the gas pressure is equal to $p_1 = p_a + p'$ (state I). Then one can read from the manometer the fluid height difference h_1, which corresponds to the difference p' between the pressure in the vessel and the atmospheric pressure. In the next step the valve K_1 is opened to let the air escape from the vessel. The air decompresses adiabatically and its temperature decreases to a value $T_2 < T_o$ (state II). The valve K_1 should be close immediately after the fluid levels in the manometer have equalized. The air in the vessel warms up to the ambient temperature T_o within several minutes and its pressure reaches the value $p_2 = p_a + p''$, lower than p_1 (state III). The value p'' corresponds to the fluid height difference in the U-tube, h_2. Inserting both values to the equation (2) the value of κ can be computed. The measurement should be repeated a few times during the experiment for greater accuracy.

5. Sequence of actions

1. Check if the amount of fluid in the U-tube is sufficient to carry out the experiment, with open valves K_1 and K_2.
2. Close the valve K_1 and increase the pressure in the vessel using the air pump, so that the fluid height difference in the manometer is about 5 – 6 cm. **Warning:** Take care when using the pump so as not to blow the fluid out of the manometer tube.
3. Close the valve K_2 and wait 5 - 6 minutes until the temperature in the vessel equalizes with the ambient temperature. Read the fluid height difference h_1 from the manometer and write it down.
4. Open the valve K_1 to decompress the gas (by letting it escape from the vessel) but close the valve immediately when the fluid levels in the manometer equalize. The pressure in the vessel is then equal to the atmospheric pressure p_a.

5. Wait 4 - 6 minutes letting the gas in the vessel warm up to the ambient temperature and read the fluid height difference h_2.
6. Calculate the value of κ using the expression (2).
7. Repeat steps 2 ÷ 7 about 6 - 10 times.
8. Collect results of the measurements and calculations in a table:

h_1 [cm]	h_2 [cm]	κ

6. Structure of the report

1. Short description of the method for determining the c_p/c_V factor.
2. Table with measurement results.
3. Calculation of errors:
   a) Mean value $\bar{\kappa}$ computed from the values written in the table
   b) Error of the mean $\Delta\kappa$ computed according to Student's distribution, with confidence factor of 0.95.
4. Final result in the form:

$$\kappa = \bar{\kappa} \pm \Delta\kappa$$

5. Discussion of the results with special attention paid to the sources of experimental errors. Comparison of the obtained value with theoretical predictions.

7. References

[1] R. Resnick, D.Holliday, Fizyka, t.1, PWN, Warszawa, 1997.
[2] B. Jaworski, A. Dietłaf, L. Miłkowska, Kurs Fizyki,t.1., PWN, 1984.
[3] J. Karniewicz, T. Sokołowski, Podstawy fizyki laboratoryjnej, skrypt PŁ, Łódź, 1996.
[4] H. Szydłowski, Pracownia fizyczna, PWN, Warszawa 1989.