Lodz University of Technology
Institute of Physics

Laboratory of electronics

Exercise E11IFE

Active integrator and differentiator circuits

Version 1.0 (3 April 2024)

Table of contents:

Before you start to perform an experiment you are obliged to have mastered the following theoretical subjects:

1. Properties of an ideal and real operational amplifier, including parameters describing the limited operating speed and the input offset voltage of the amplifier. [1, 2, 4, 5]
2. Principle of operation of active RC integrator and differentiator circuits. [2, 3]
3. Advantages of active integrator and differentiator circuits over passive RC circuits. [3, 4]

1. Purpose of the exercise

The purposes of experiment are:

1. Investigating the properties of active RC integrator and differentiator circuits.
2. Getting to know the limitations occurring in real active RC integrator and differentiator circuits.

2. Hazards

Type	Absence	Low	Medium	High
electrical radiation hazards		+		
optical radiation hazards	+			
mechanical hazards (including acoustic hazards, noise)	+			
electromagnetic radiation hazards (invisible)	+			
biological hazards	+			
ionizing radiation hazards	+			
chemical hazards	+			
thermal hazards (including explosion and fire)	+			

The cables with banana plugs are designed exclusively for use in low-voltage circuits – do not connect them to the mains supply 230 V.

3. Introduction

3.1. Active integrator circuit

Let us consider the active integrator circuit shown in Fig. 1, assuming that the operational amplifier (op-amp) is ideal (the parameters of the ideal and real amplifier are given in the instruction for exercise E08 “Operational amplifier”). Because the input resistance of an ideal amplifier is infinite, the current flowing through the resistor Z_1 is equal to the current i flowing through the capacitor Z_2. Moreover, the infinite voltage gain of an ideal op-amp causes the potential of the “–” input to be almost equal to the potential of the “+” input connected to ground. It follows that the voltage on the resistor Z_1 is equal to the input voltage u_{in} and the voltage on the capacitor Z_2 is equal to the output voltage u_{out}.

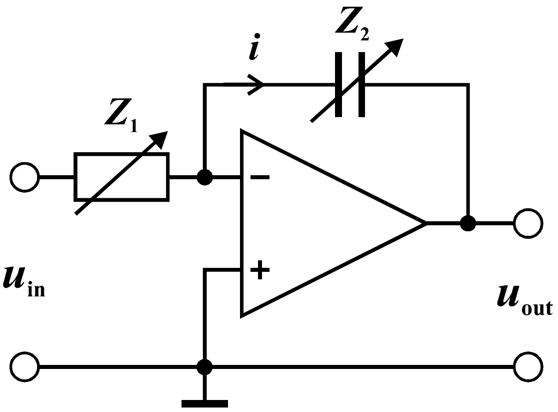

Fig. 1. Circuit diagram of an ideal active integrator.

Under the above assumptions, Ohm’s Law written for the resistor Z_1 takes the following form

$$u_{\text{in}} = Z_1\, i. \tag{1}$$

Using the definition of electric capacitance, we can write the change in electric charge dq on the capacitance Z_2 as

$$dq = Z_2\, du_{\text{out}}\,. \tag{2}$$

The change in electric charge dq can also be expressed in terms of the time dt and the current i

$$dq = -i\, dt, \tag{3}$$

where the sign “–” results from the direction of the current i assumed in Fig. 1. If the opposite direction of the current was assumed, the sign “–” would appear in equation (1). The above system of equations (1)-(3) leads to the following relationship between the output and input voltages of the integrator circuit

$$\frac{du_{\text{out}}}{dt} = -\frac{u_{\text{in}}}{Z_1 Z_2}\,. \tag{4}$$

The above differential equation takes the following form after integration:

$$u_{\text{out}}(t) - u_{\text{out}}(0) = -\frac{1}{Z_1 Z_2}\int_0^t u_{\text{in}}\, dt\,. \tag{5}$$

Response of the integrator circuit to a sinusoidal waveform

Let us assume a sinusoidal input waveform

$$u_{\text{in}}(t) = U_0 \sin(2\pi f\, t). \tag{6}$$

Substituting formula (6) into (5) and assuming $u_{\text{out}}(0) = 0$, we obtain the following circuit response:

$$u_{\text{out}}(t) = \frac{U_0}{2\pi f\, Z_1 Z_2}\cos(2\pi f\, t) = \frac{U_0}{2\pi f\, Z_1 Z_2}\sin(2\pi f\, t + \pi/2)\,. \tag{7}$$

As can be seen from equation (7), the output waveform is also sinusoidal, but its phase is shifted by +90° relative to the input signal and its amplitude $U_0/(2\pi f\, Z_1 Z_2)$ is inversely proportional to the frequency f and the time constant $Z_1 Z_2$.

Response of the integrator circuit to a triangle waveform

Let us consider one period of a triangular waveform with amplitude U_0 written for the time interval $-T/4...+3T/4$

$$u_{\text{in}}(t) = \begin{cases} 4U_0 f\, t, & \text{for } -\tfrac{1}{4}T \le t \le \tfrac{1}{4}T, \\ -4U_0 f\, t + 2U_0, & \text{for } \tfrac{1}{4}T \le t \le \tfrac{3}{4}T. \end{cases} \tag{8}$$

After substituting formula (8) into (5) and choosing such value of $u_{\text{out}}(0)$ that the output waveform contain no DC component, we obtain

$$u_{\text{out}}(t) = \begin{cases} \dfrac{U_0}{8Z_1 Z_2}(-16 f\, t^2 + T), & \text{for } -\tfrac{1}{4}T \le t \le \tfrac{1}{4}T, \\ \dfrac{U_0}{8Z_1 Z_2}(16 f\, t^2 - 16t + 3T), & \text{for } \tfrac{1}{4}T \le t \le \tfrac{3}{4}T. \end{cases} \tag{9}$$

The waveform (9) consists of fragments of parabolas with arms directed alternately downwards or upwards and passes through zero at the moments $t = T/4$ and $t = 3T/4$. This waveform resembles a sine wave, but it also contains odd harmonics and the level of harmonic distortion is about 5 %. The output voltage reaches extreme values $U_{\text{out,max}}$ and $U_{\text{out,min}}$ at $t = 0$ and $t = T/2$, respectively, which are given by

$$U_{\text{out,max}} = \frac{U_0 T}{8Z_1 Z_2}, \qquad U_{\text{out,min}} = -\frac{U_0 T}{8Z_1 Z_2}\,. \tag{10}$$

Response of the integrator circuit to a square waveform

Let us consider one period of a square waveform with amplitude U_0 written for the time interval $0...T$

$$u_{\text{in}}(t) = \begin{cases} U_0, & \text{for } 0 \le t < \tfrac{1}{2}T, \\ -U_0, & \text{for } \tfrac{1}{2}T \le t < T. \end{cases} \tag{11}$$

After substituting formula (11) into (5) and choosing such value of $u_{\mathrm{out}}(0)$ that the output waveform contain no DC component, we obtain

$$u_{\mathrm{out}}(t)=\begin{cases}-\dfrac{U_0}{Z_1Z_2}\left(t-\dfrac{1}{4}T\right), & \text{for } 0\le t<\tfrac{1}{2}T,\\ \dfrac{U_0}{Z_1Z_2}\left(t-\dfrac{3}{4}T\right), & \text{for } \tfrac{1}{2}T\le t<T.\end{cases} \tag{12}$$

Formula (12) describes a triangular waveform that passes through zero at the moments $t = T/4$ and $t = 3T/4$. The output voltage reaches extreme values $U_{\mathrm{out,max}}$ and $U_{\mathrm{out,min}}$ at $t = 0$ and $t = T/2$, respectively, which are given by

$$U_{\mathrm{out,max}}=\frac{U_0T}{4Z_1Z_2}, \qquad U_{\mathrm{out,min}}=-\frac{U_0T}{4Z_1Z_2}. \tag{13}$$

3.2. Active differentiator circuit

When considering an ideal op-amp, we obtain analogously as in section 3.1, that the same current i flows through the capacitor Z_1 and the resistor Z_2 in the circuit shown in Fig. 2. Moreover, the voltage on the capacitor Z_1 is equal to the input voltage u_{in} and the voltage on the resistor Z_2 is equal to the output voltage u_{out}.

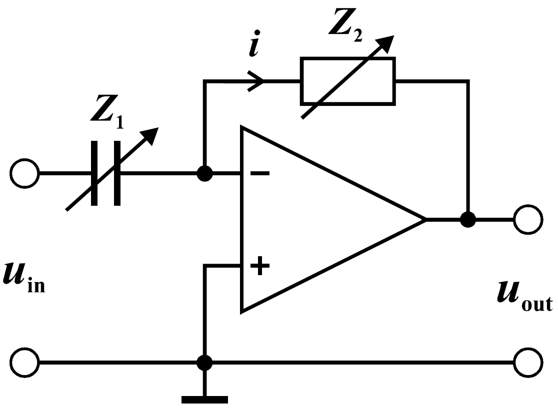

Fig. 2. Circuit diagram of an ideal active differentiator.

Hence, Ohm's law written for the Z_2 resistor takes the following form

$$u_{\mathrm{out}} = -Z_2\, i. \tag{14}$$

Using the definition of electric capacitance, we can write the change in electric charge dq on the capacitance Z_1 as

$$dq = Z_1\, du_{\mathrm{in}}\,. \tag{15}$$

This change dq can also be expressed in terms of the time dt and the current i

$$dq = i\, dt. \tag{16}$$

The system of equations (14)-(16) leads to the following relationship between the output and input voltages of the differentiator circuit

$$u_{\text{out}}(t) = -Z_1 Z_2 \frac{du_{\text{in}}}{dt} \,. \tag{17}$$

Response of the differentiator circuit to a sinusoidal waveform

Substituting the sinusoidal input voltage given by formula (6) into (17) yields

$$u_{\text{out}}(t) = -2\pi f\ Z_1 Z_2\, U_0 \cos(2\pi f\, t) = 2\pi f\ Z_1 Z_2\, U_0 \sin(2\pi f\, t - \pi/2)\,. \tag{18}$$

Formula (18) shows that the output waveform is also sinusoidal, but its phase is shifted by −90° relative to the input signal and its amplitude $2\pi f\, Z_1 Z_2 U_0$ is proportional to the frequency f and the time constant $Z_1 Z_2$.

Response of the differentiator circuit to a triangle waveform

After substituting the triangular input signal given by formula (8) into (17), we obtain

$$u_{\text{out}}(t) = \begin{cases} -4U_0\, f\, Z_1 Z_2, & \text{dla } -\frac{1}{4}T \le t \le \frac{1}{4}T, \\ \ \ 4U_0\, f\, Z_1 Z_2, & \text{dla } \frac{1}{4}T \le t \le \frac{3}{4}T. \end{cases} \tag{19}$$

The above formula describes a square waveform with an amplitude of $4U_0 f\, Z_1 Z_2$, in which switching occurs at moments $t = T/4$ and $t = 3T/4$, i.e. at moments when the input signal changes from increasing to decreasing or vice versa.

Response of the differentiator circuit to a square waveform

Substituting the square input waveform described by formula (11) into the time derivative in equation (17) leads to a response in the form of infinitely high pulses of infinitely short duration. The negative voltage pulse appears when the input voltage is switched from $-U_0$ to $+U_0$, e.g. at $t = 0$, and the positive voltage pulse occurs when the input voltage changes in the opposite direction, e.g. at $t = T/2$. In real differentiation circuits, the height of the pulse is limited by the voltage supplying the op-amp, while the duration and shape of the pulse result from the speed of the op-amp and the quality of the passive components used.

3.3. Practical implementation of active integrator and differentiator circuits

The integrator circuit shown in Fig. 1 is exposed to unlimited accumulation of the DC component present in the input signal, which leads to saturation of the op-amp output after a sufficiently long time. The input DC voltage may come from both a function generator connected to the input of the integrator circuit and from the input offset voltage of the op-amp used. To prevent saturation, an additional resistor R_S is required in practice, which is connected in parallel with the capacitor Z_2 as shown in Fig. 3. In such a circuit, the voltage gain for DC voltage has a limited value given by

$$k_{U_0} = -\frac{R_S}{Z_1}. \tag{20}$$

If the DC component of the input voltage can reach the maximum value $U_{0,\mathrm{in}}$, then the resistance R_S should be selected so that the maximum output DC voltage lies within the range of available output voltages with a significant margin

$$U_{\mathrm{out,min}} << k_{U_0} U_{0,\mathrm{in}} << U_{\mathrm{out,max}}. \tag{21}$$

Moreover, if we expect that the modified circuit will closely approximate the ideal integrating circuit, another condition must also be met

$$R_S Z_2 >> 1/f_{\mathrm{min}}, \tag{22}$$

where f_{min} is the lowest frequency of the useful component in the processed waveform.

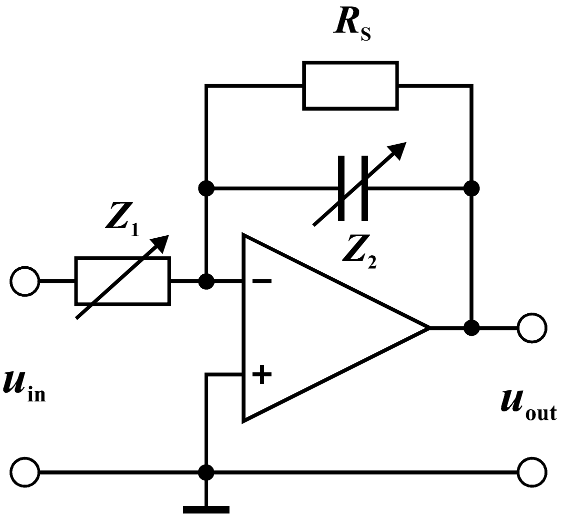

Fig. 3. Circuit diagram of an active integrator with gain correction for the DC and low-frequency signals.

In practice, the differentiator connected according to the diagram shown in Fig. 2 causes problems in the high frequency range. Because a real op-amp is characterized by a finite signal propagation time and a limited rate of change in the output voltage, a sudden change in the input voltage causes short-term operation of the amplifier without a negative feedback loop, which may result in excitation of decaying or permanent oscillations. To suppress these oscillations, the high-frequency voltage gain should be reduced, e.g. by adding a series resistor R^* as shown in Fig. 4. If we expect that the modified circuit will closely approximate the ideal differentiating circuit, the following condition must also be met

$$R^* Z_1 \ll 1/f_{\max}, \tag{23}$$

where $f_{\max}$ is the highest frequency of the useful component in the processed waveform. In addition, cable lengths should be minimized, the op-amp chosen should be fast enough, and smoothing capacitors should be mounted very close to the amplifier's power supply terminals.

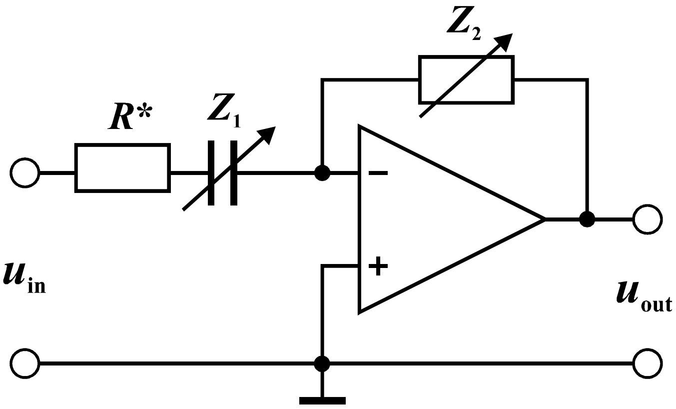

Fig. 4. Circuit diagram of an active differentiator with correction for high-frequency (fast-changing) signals.

4. Available equipment

4.1. Experimental module

The front panel of experimental module is shown in Fig. 5. The module is composed of an operational amplifier and sets of switched resistors and capacitors that can be used at the inverting "–" input of the amplifier (switch Z_1), as negative feedback (Z_2), as positive feedback (R_3), and as a load on the amplifier output (R_L).

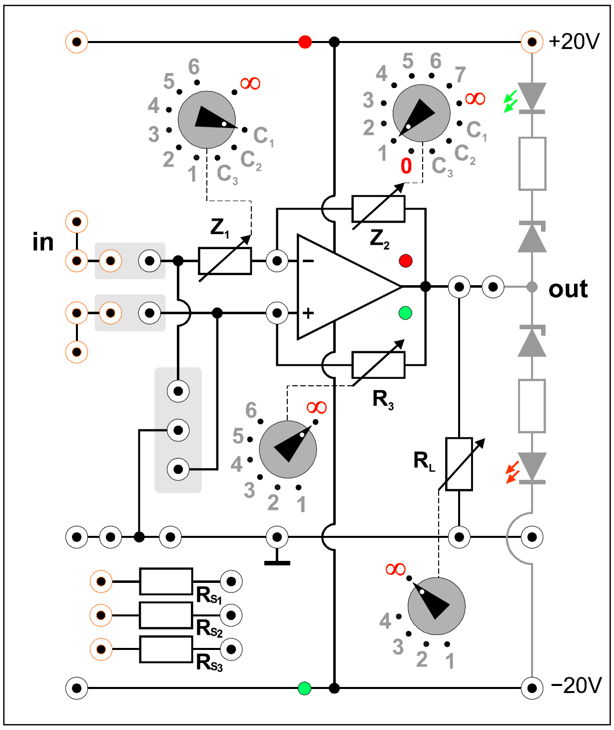

Fig. 5. The front panel of the experimental module.

4.2. Power supply

To supply power to the experimental module, the DC laboratory power supply SIGLENT model SPD3303D is used [6]. Before the measurements, the power supply must be connected to the experimental module and the serial operation mode should be selected. After switching on the power, the symmetrical supply voltage at ±20 V must be set.

4.3. Function generator

The function generator DF1641B [6].

4.4. Oscilloscope

In this exercise, a two-channel SIGLENT SDS1052DL digital oscilloscope is used [6]. This oscilloscope allows to save the oscillogram on an external USB memory stick as a BMP file. To save the image, press the PRINT button. Files with subsequent images are automatically named as SDS00001.BMP, SDS00002.BMP,… . The saved waveform images can be used instead of manual drawing waveforms. Students who wish to use this option should bring their own USB memory stick with a FAT32 partition.

5. Experimental procedure

5.1. Investigation of the active integrator circuit

1. Connect the circuit according to the diagrams shown in Figs. 6 and 7. Set the Z_1 switch to the “1” position, the Z_2 switch to the “C_1” position, and the R_3 and R_L switches to the “∞” position. Connect the R_{S2} resistor between the “−” input and the output of the op-amp. Use only the universal OUTPUT in the function generator FG. **Put the generator’s amplitude control knob in the extreme left position. Do not turn on the power supply outputs yet.**
2. After the supervisor has checked the circuit, turn on the devices. Press the SER button to set the serial mode of the laboratory power supply, adjust the symmetrical voltage to ±20 V, and turn on the outputs of the MASTER and SLAVE channels. Check out the status of the red and green LED indicators on the +20 V and –20 V lines in the experimental module.
3. Make sure that all LED indicators in the COUNTER section on the front panel of the function generator are off. Select a sinusoidal waveform, set the frequency to 100 Hz, set the output amplitude range $(0.2 \div 2)\ \mathrm{V_{p\text{-}p}}$ and initially set the signal amplitude to $U_{\mathrm{in,max}} = 0.5\ \mathrm{V}$ following the generator’s display showing the peak-to-peak voltage $\mathrm{V_{p\text{-}p}}$ (remember to difference between amplitude and peak-to-peak voltage).
4. Press the DEFAULT SETUP button to restore the oscilloscope to its default settings. Set the oscilloscope to dual-channel mode (both CH1 and CH2 buttons are illuminated) with DC coupling mode in both channels. Adjust the optimal parameters for displaying both waveforms on the oscilloscope screen.
5. Make sure the op-amp output does not reach saturation levels at 100 Hz. If distortions of the output sine wave are visible, reduce the amplitude of the generator signal as much as necessary.
6. Measure the input amplitude $U_{\mathrm{in,max}}$, the output amplitude $U_{\mathrm{out,max}}$ and the phase shift φ of the output voltage relative to the voltage at the input of the integrator circuit as a function of frequency f. Unless the supervisor advises otherwise, select several frequencies in the range $(100 \div 2000)$ Hz. When changing the frequency, remember to adjust the oscilloscope settings to obtain the optimal waveforms display. Record the measurement results in Table 1. Draw or save several oscillograms of the input and output waveforms for some selected characteristic situations along with the oscilloscope settings [V/DIV] and [s/DIV].
   Tip 1: The $U_{\mathrm{in,max}}$ and $U_{\mathrm{out,max}}$ amplitude values can be read from the oscilloscope screen in numerical form by pressing the MEASURE button and then using the buttons on the right side of the screen to select the peak-to-peak voltages Vpp for both CH1 and CH2 channels. Calculate the amplitudes as half of the corresponding peak-to-peak voltages.
   WARNING: the function generator may produce a waveform with a small DC component even when DC offset is disabled on the generator control panel. Because the integrator circuit amplifies the DC voltage many times stronger than the AC voltage, the amplitude should be read from the oscilloscope as Vpp/2, while all voltage readings relative to the 0 V level (e.g. Vmax, Vmin, Vtop,…) may give incorrect results. Do not use the AC coupling mode on the oscilloscope because it may significantly distort the observed waveforms at the lowest frequencies.
   Tip 2: to display the phase difference CH1-CH2 on the oscilloscope screen, open the MEASURE menu, select the Delay group and then select Type Phase.

7. Repeat the measurements described in points 5 and 6 for a triangular and square waveforms at the input of the integrator circuit. The measurements of the phase shift φ should only be made for a sinusoidal waveform.
8. Turn off the power and disconnect the circuit (except the power supply cables).

Table 1. The measurement and observation results obtained for the active integrator circuit.

Frequency	Circuit settings		Input waveform		Output waveform		Phase shift
f [Hz]	Z_1 [kΩ]	Z_2 [μF]	$U_{in,max}$ [V]	shape	$U_{out,max}$ [V]	shape	φ [deg]

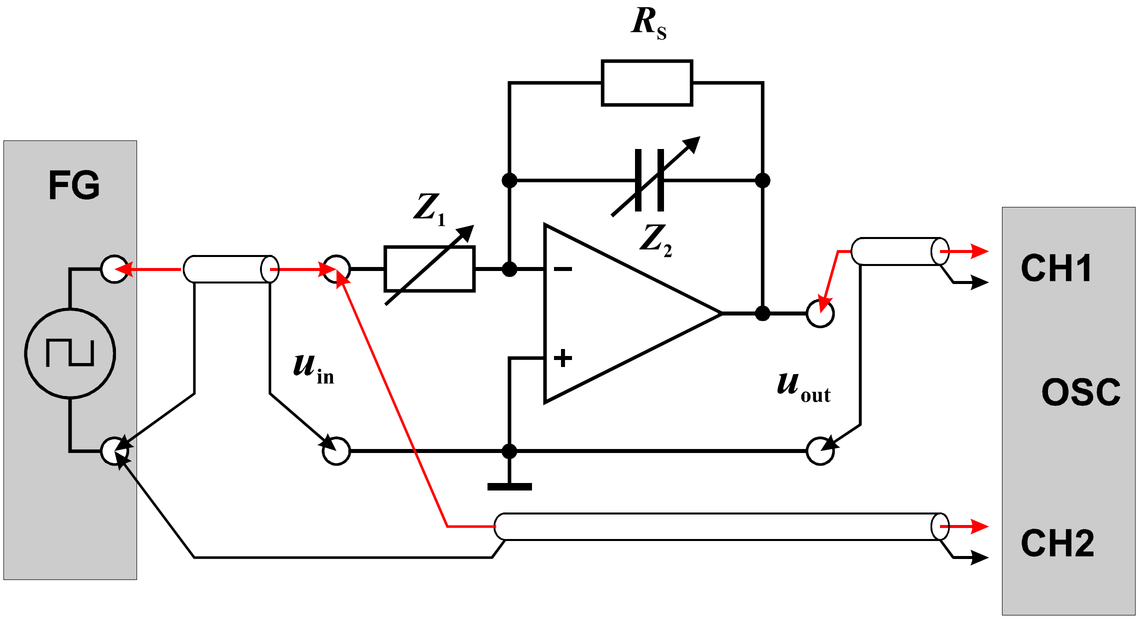

Fig. 6. Scheme of connection diagram for testing the active integrator circuit.

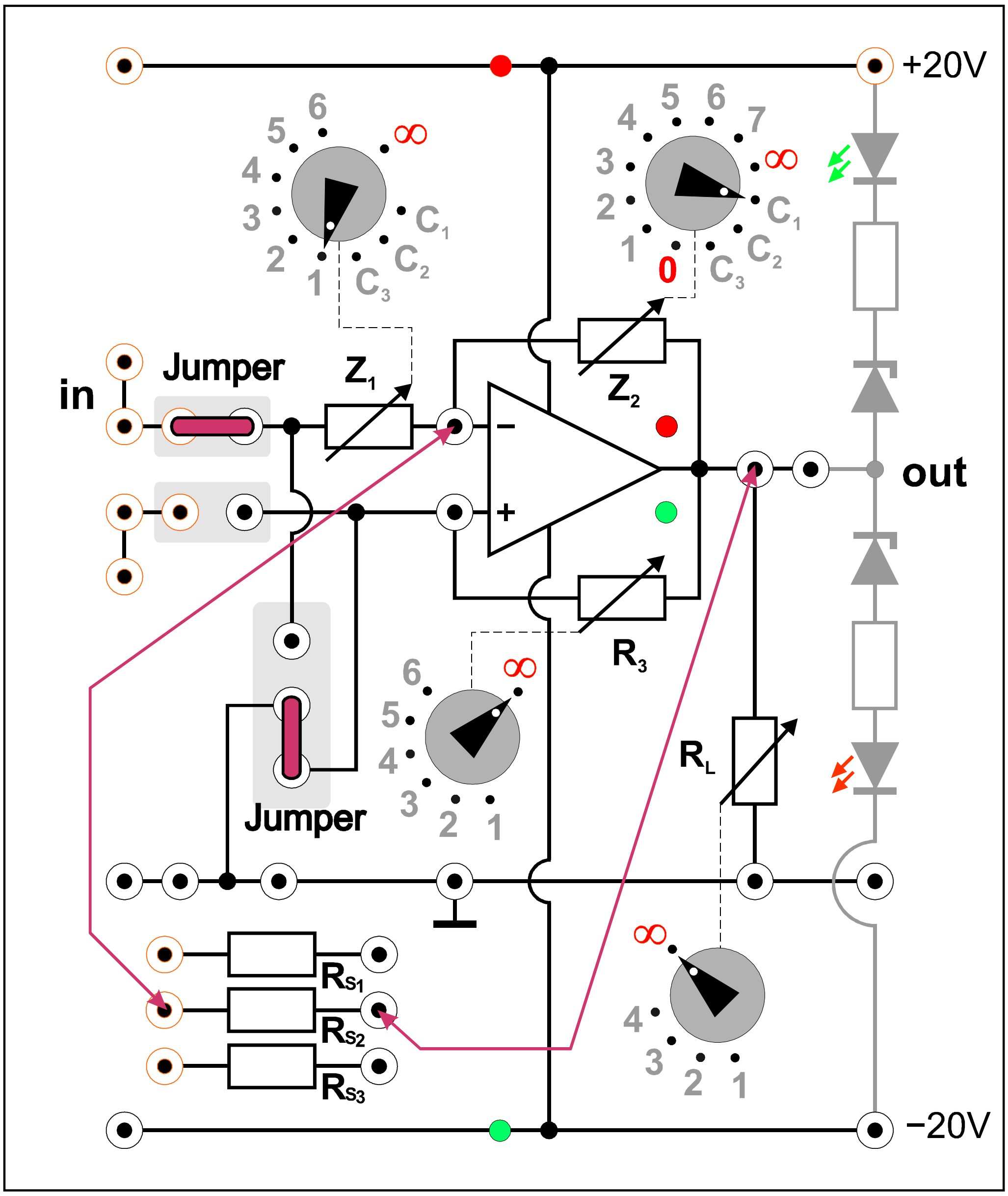

Fig. 7. The active integrator circuit built using the experimental module.

5.2. Investigation of the active differentiator circuit

1. Connect the circuit according to the diagrams shown in Figs. 8 and 9. Set the Z_1 switch to the "C_1" position, the Z_2 switch to the "1" position, and the R_3 and R_L switches to the "∞" position. Connect a cable with a built-in 50 Ω resistor between the input "in" of the circuit and the impedance Z_1 (if there is no cable marked "50", ask the laboratory technical service). **Put the generator's amplitude control knob in the extreme left position. Do not turn on the power supply outputs yet.**
2. After the supervisor has checked the circuit, turn on the devices.
3. Select a sinusoidal waveform in the function generator, set the frequency to 2 kHz, set the output amplitude range 20 mV$_{\text{p-p}}$ ÷ 0.2 V$_{\text{p-p}}$ and initially set the signal amplitude to $U_{\text{in,max}}$ = 50 mV following the generator's display showing the peak-to-peak voltage V$_{\text{p-p}}$.
4. Set the oscilloscope to dual-channel mode (both CH1 and CH2 buttons are illuminated) with DC coupling mode in both channels. Adjust the optimal parameters for displaying both waveforms on the oscilloscope screen.
5. Make sure the op-amp output does not reach saturation levels at 2 kHz. If distortions of the output sine wave are visible, reduce the amplitude of the generator signal as much as necessary.
6. Measure the input amplitude $U_{\text{in,max}}$, the output amplitude $U_{\text{out,max}}$ and the phase shift φ of the output voltage relative to the voltage at the input of the differentiator circuit as a function of frequency *f*. Unless the supervisor advises otherwise, select several frequencies in the range (100 ÷ 2000) Hz. Record the measurement results in Table 2. Draw or save several oscillograms of the input and output waveforms for some selected characteristic situations along with the oscilloscope settings [V/DIV] and [s/DIV].

   WARNING: in the case of a differentiator circuit, the DC component should not be significant in the observed waveforms, but there may be strong noise and decaying oscillations that distort amplitude measurements using the oscilloscope in the Vmax, Vmin and Vpp modes. These distortions may also appear at the input of the tested circuit. Therefore, it is recommended to read the amplitudes in both channels from the MEASURE menu as Vtop voltages, i.e. as a flat peak voltage (omitting narrow peaks), or as half of the Vamp voltage (the Vamp symbol does not mean the amplitude, but the difference between the flat peak voltage and the flat base voltage, omitting narrow peaks that are included in Vpp readings).
7. Repeat the measurements described in points 5 and 6 for a triangular and square waveforms at the input of the differentiator circuit. The measurements of the phase shift φ should only be made for a sinusoidal waveform.

   WARNING: when measuring the height of output voltage peaks that correspond to a square wave at the input, the Vmax mode in the CH1 channel of the oscilloscope should be selected in the MEASURE menu.
8. Turn off the power and disconnect the circuit.

Table 2. The measurement and observation results obtained for the active differentiator circuit.

Frequency	Circuit settings		Input waveform		Output waveform		Phase shift
f [Hz]	Z_1 [µF]	Z_2 [kΩ]	$U_{\text{in,max}}$ [V]	shape	$U_{\text{out,max}}$ [V]	shape	φ [deg]

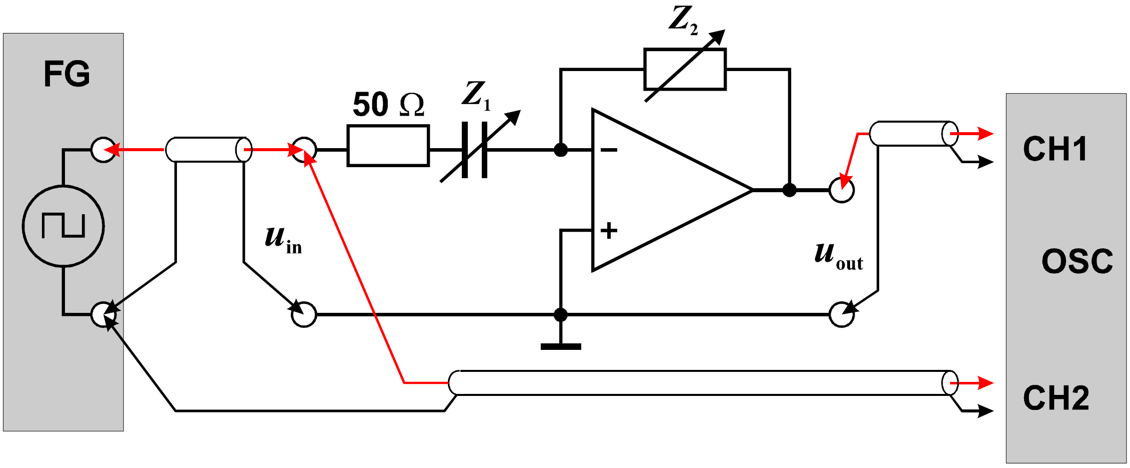

Fig. 8. Scheme of connection diagram for testing the active differentiator circuit.

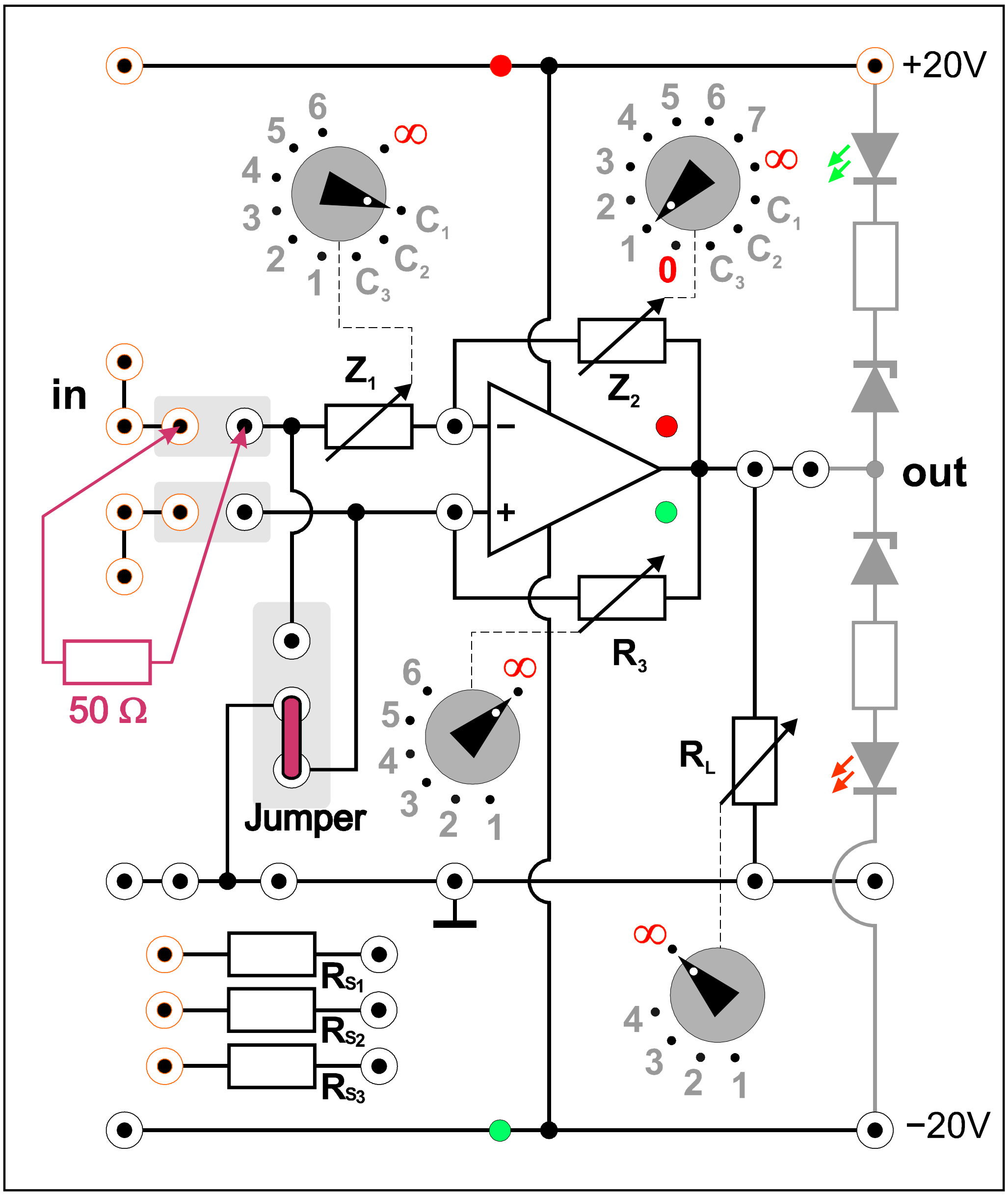

Fig. 9. The active differentiator circuit built using the experimental module.

6. Report elaboration

Report has to be composed of:

1. Front page (by using a pattern).
2. Description of experiment purposes.
3. List of used instruments and devices (id/stock number, type, setting and range values).
4. Schematic diagram of tested circuit.
   The report should contain only diagrams of the circuits, which were actually compiled during the measurements. Each scheme must be accompanied by a sequence number and title. All the components shown in the diagram must be clearly described and identified using commonly used symbols.
5. The results of measurements and observations written in Tables 1 and 2, and the results of calculation along with the formulas used (without derivations). In particular, the following calculations should be done:
   - 5.1. In the case of an integrator circuit, calculate the theoretical values of the output amplitudes according to formulas (7), (10) and (13) for a sinusoidal, triangular and square waveform at the input, respectively.
   - 5.2. In the case of a differentiator circuit, calculate the theoretical values of the output amplitudes according to formulas (18) and (19) for a sinusoidal and triangular waveform at the input, respectively.
6. Oscillograms and analysis. In particular:
   - 6.1. Include sample oscillograms of the waveforms at the input and output of the integrator circuit along with a description of the oscilloscope settings and the settings of the R and C elements in the experimental module. Compare the observed transformations of waveforms with theoretical predictions. Assess whether the measured output amplitudes are consistent with theoretical values. In the case of a sinusoidal waveform, also compare the measured and theoretical values of the phase shift φ.
   - 6.2. Include sample oscillograms of the waveforms at the input and output of the differentiator circuit along with appropriate descriptions. Compare the observed transformations of waveforms with theoretical predictions. In the case of a sinusoidal and triangular input waveforms, assess whether the measured output amplitudes are consistent with the theoretical values. For a sinusoidal waveform, also compare the measured and theoretical values of the phase shift φ.
7. Remarks and final conclusions.

The completeness, correctness, clarity of presentation of the results (in the form of tables, oscillograms and calculations together with descriptions) and the quality of discourse and conclusions will all be evaluated. Theoretical introduction is not required and is not included in the assessment.

7. References

7.1. Basic reference materials

[1] A. Chwaleba, B. Moeschke, *Pracownia elektroniczna. Część 2, układy elektroniczne*, Wydawnictwa Szkolne i Pedagogiczne, Warszawa 1980.

[2] S. Kuta, *Elementy i układy elektroniczne. Część 1*, Uczelniane Wydawnictwa Naukowo-Dydaktyczne Akademii Górniczo-Hutniczej, Kraków 2000.

[3] Z. Kulka, M. Nadachowski, *Liniowe układy scalone i ich zastosowanie*, WKiŁ, Warszawa 1977.

[4] P. Górecki, *Wzmacniacze operacyjne*, BTC, Warszawa 2004.

[5] M. Łakomy, J. Zabrodzki, *Liniowe układy scalone w technice cyfrowej*, PWN, Warszawa 1987.

7.2. Other reference materials

[6] User manuals for power supply, function generator, and oscilloscopes available on the website: https://fizyka.p.lodz.pl/pl/dla-studentow/information-technology/fundamentals-of-electronics/

8. Appendixes

A1. Tables of resistances and capacitances

Z_1

Position	Value
1	5 kΩ
2	10 kΩ
3	15 kΩ
4	20 kΩ
5	25 kΩ
6	30 kΩ
∞	∞ Ω
C_1	0.1 µF
C_2	1.0 µF
C_3	10 µF

Z_2

Position	Value
0	0 Ω
1	10 kΩ
2	20 kΩ
3	50 kΩ
4	100 kΩ
5	200 kΩ
6	500 kΩ
7	1 MΩ
∞	∞ Ω
C_1	0.1 µF
C_2	1.0 µF
C_3	10 µF

R_3

Position	Value
1	20 kΩ
2	50 kΩ
3	100 kΩ
4	200 kΩ
5	500 kΩ
6	1 MΩ
∞	∞ Ω

R_L

Position	Value
1	2 kΩ
2	5 kΩ
3	10 kΩ
4	20 kΩ
∞	∞ Ω

R_S

Position	Value
R_{S1}	10 kΩ
R_{S2}	300 kΩ
R_{S3}	10 MΩ