

EXERCISE 299

Measuring relation between the water boiling temperature and the external pressure

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March 12, 2009

Abstract

The report presents the measurements of the water boiling temperature as a function of the pressure. The relation has been determined by heating the water under reduced pressure. The experimental results allows to compute the specific heat of water vaporisation. The obtained value is in good agreement with the one published in textbooks.

1 Introduction

The aim of this exercise was a measurement of a relation between the water boiling temperature and the external pressure. The phenomenon of boiling can take place in any temperature, in which the liquid state can exist, i. e. between the triple and critical points of the substance. It takes place when the vapour pressure is equal to the external pressure, which results in the evaporating phenomena taking place in the whole volume of the liquid. This means that the boiling temperature is dependent on the external pressure. For water, the common boiling temperature is assumed to be 100°C. In a matter of fact it is equal to this value only in so called normal conditions, i. e. under the pressure of 101.325 kPa. In other cases the water boiling temperature can significantly differ from 100°C. In this exercise we determine these differences. Furthermore, using the measured relation between the water boiling temperature and the external pressure, we determine the specific heat of vaporisation of water.

The report consists of three parts. In the beginning we present the basic model of the analysed phenomenon. We show the schematic diagram of the laboratory set-up and shortly present the method of measurements. Next we present our experimental results and their analysis. In the end we conclude.

2 Theory and measurement method

As already stated in the introduction, the boiling temperature of any liquid depends on the external pressure. Their mutual relation—under assumption that the vapour is a perfect gas and liquid state occupies no volume—is described by the Clausius-Clapeyron equation [1, 2]

$$\ln \left(\frac{p_i}{p_0} \right) = -\frac{\lambda_p}{R} \left(\frac{1}{T_i} - \frac{1}{T_0} \right), \quad (1)$$

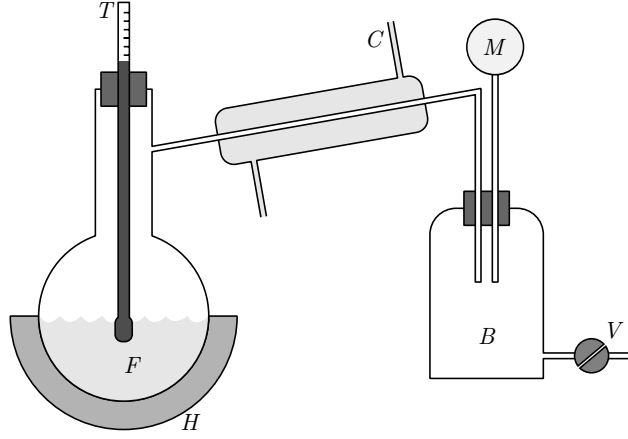


Figure 1: Diagram of the experimental set-up. It consists of a flask F containing analysed liquid, a heater H , thermometer T , cooler C , big bottle B used for ensuring constant pressure inside a flask, a valve V and manometer M .

where T_i is the liquid boiling temperature under the pressure p_i , $R = 8,31447 \frac{\text{J}}{\text{molK}}$ is the universal gas constant, λ_p is a specific heat of vaporisation, T_0 is boiling temperature under normal conditions, $p_0 = 101,325 \text{ kPa}$ states for atmospheric pressure in normal conditions. Using the above relation we can write

$$T_i = \left[-\frac{R(\ln \log p_i - \ln p_0)}{\lambda_p} + \frac{1}{T_0} \right]^{-1}, \quad (2)$$

which means that the reciprocity of the boiling temperature T_i^{-1} is proportional to the natural logarithm of the pressure $\ln p_i$, i. e.

$$T_i^{-1} = a \ln p_i + b, \quad (3)$$

where

$$a = -R/\lambda_p, \quad (4)$$

$$b = -\frac{R}{\lambda_p} \ln p_0 + \frac{1}{T_0}. \quad (5)$$

In consequence, knowing the coefficients a and b it is possible to determine the specific heat of vaporisation λ_p , using Eqs. (3) and (4) transformed into

$$\lambda_p = -R/a. \quad (6)$$

The experiment is performed with an apparatus shown in Fig. 1. It consists of a heated flask containing analysed water, a cooler and elements necessary for adjusting the air pressure. In the beginning of the experiment the air pump was used to twice decrease the pressure inside of the flask. Next, the water was heated up until it began to boil. In this moment its temperature was measured with the attached thermometer. In the next steps we were letting some amount of air into the system each time, in order to increase the pressure a little, and were measuring the boiling temperature. This procedure was performed until we have reached again the atmospheric pressure inside the flask.

Table 1: Measured results: quantities read from the manometer and thermometer h_i and T_i , respectively, and calculated quantities p_i , $\ln p_i$ and T_i^{-1} .

h_i [kPa]	p_i [kPa]	T_i [°C]	T_i [K]	$\ln p_i$	T_i^{-1} [10^{-3} K^{-1}]
71.4	29.0	71	344	3.368	2.907
62.0	38.4	77	350	3.648	2.857
51.6	48.8	83	356	3.888	2.811
46.2	54.2	84	357	3.993	2.801
37.2	63.2	86	359	4.147	2.788
25.7	74.7	90	364	4.314	2.750
19.2	81.2	91	365	4.398	2.743
16.1	84.3	97	371	4.434	2.699
0.0	100.4	100	373	4.609	2.679

3 Results

The results of our measurements are presented in Table 1. It contains the values read from the manometer and boiling temperatures, as well as computed quantities: pressure inside the flask and functions T_i^{-1} and $\ln p_i$. The pressure in the system was determined using the relation

$$p_i = p_b - h_i, \quad (7)$$

where h_i is the manometer reading and p_b is the atmospheric pressure, in the day of experiment equal to $p_b = (100.4 \pm 0.1)$ kPa, as read from the wall-barometer in the laboratory.

The uncertainties of the pressure and boiling temperature were $\Delta p_i = 0,1$ kPa and $\Delta T_i = 2$ K, respectively. The latter value was estimated rather high due to the difficulty of precise observation of the moment in which the boiling begins.

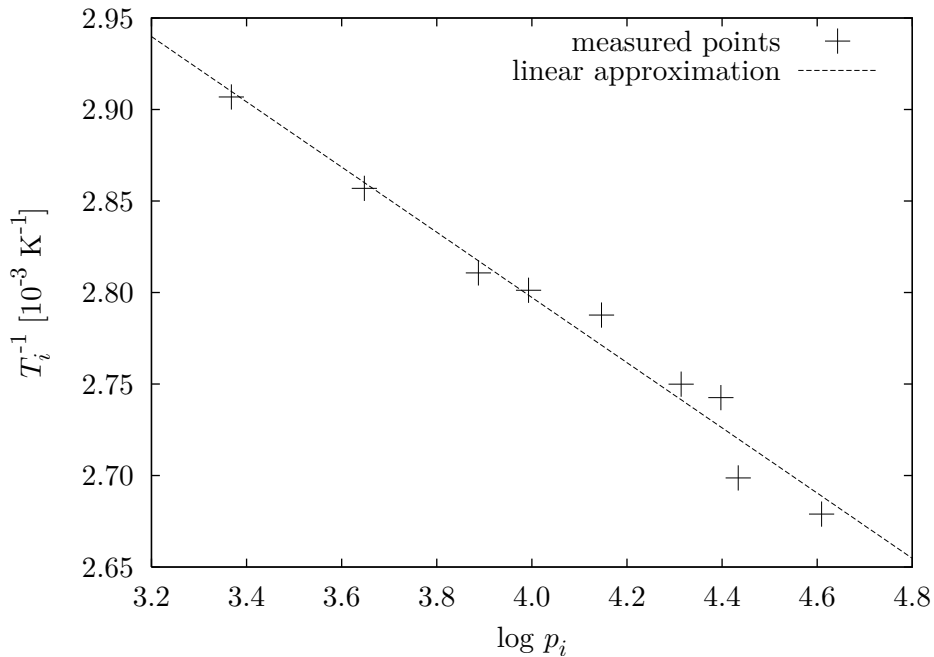


Figure 2: Plot of T_i^{-1} as a function of $\ln p_i$ and the linear function approximating the measurements with the best possible accuracy.

The results are plotted in Fig. 2. According to Eq. (3) the relation between T_i^{-1} and $\ln p_i$ is linear. The slope of this function can be easily determined with the least squares method and is equal to

$$a = (-1.78 \pm 0.12) \cdot 10^{-4} \text{ K}^{-1}.$$

Using this value it is possible to determine the specific heat of vaporisation from Eq. (6):

$$\lambda_p = -\frac{8,31447 \text{ J mol}^{-1} \text{ K}^{-1}}{-1.78 \cdot 10^{-4} \text{ K}^{-1}} = 46.6 \cdot 10^3 \frac{\text{J}}{\text{mol}}.$$

A more detailed analysis of Fig. 2 reveals that the error of the above quantity is influenced by some other factors than the uncertainties of the measuring devices, denoted by ΔT_i and Δp_i . Hence, the decisive factor is the uncertainty of the slope Δa , which results from the dissipation of the measured points. In consequence the uncertainty $\Delta \lambda_p$ can be computed using absolute error propagation for Eq. (6):

$$\Delta \lambda_p = \frac{R}{a^2} \Delta a, \quad (8)$$

substituting numerical values we have

$$\Delta \lambda_p = \frac{8,31447 \text{ J mol}^{-1} \text{ K}^{-1}}{(-1,78 \cdot 10^{-4} \text{ K}^{-1})^2} \cdot 0,12 \cdot 10^{-4} \text{ K}^{-1} = 3,2 \cdot 10^3 \frac{\text{J}}{\text{mol}}.$$

Finally, the determined specific heat of vaporisation for water is equal to

$$\lambda_p = (47 \pm 4) \frac{\text{kJ}}{\text{mol}}.$$

4 Conclusions

The specific heat of water vaporisation, determined in our experiment, is equal to $\lambda_p = (47 \pm 4) \frac{\text{kJ}}{\text{mol}}$. Its tabular value, reproduced from Ref. [3] is $\lambda_{p\text{tab}} = 43.630 \frac{\text{kJ}}{\text{mol}}$. Both results are in agreement withing the error bounds. This means that no big systematic errors occurred in our experiment and the influence of statistical errors was minimised due to the use of the least squares method. The measurements were in close vicinity of the theoretical expectations. Hence, one can conclude that the presented method allows to determine the specific heat of vaporisation with a few percent accuracy (in our case it is exactly 7%). In order to increase precision one should take special care to accurately determine the exact temperature of boiling and, at the same time, measure the pressure. The difficulties with the correct reading of these quantities should be considered the main source of errors in our experiment.

References

- [1] M. Bean, P. Snowwhite, *Incredible laboratory manual*, materials of Technical University of Lodz, Lodz 2009.
- [2] H. Szydłowski, *Experimental Physics*, 2nd ed., Wiley, London 1994.
- [3] A. Dumbledore, *Totally improbable physical table and plots*, Springer-Verlag, Amsterdam 2001.