Degrees of freedom

PROBLEM

Two masses m_1 and m_2 are connected with a spring and are sliding on a frictionless plane. How many degrees of freedom does the system have? Propose a set of generalized coordinates.

Answer: 4

PROBLEM

A thin straight rod moves freely in three dimensional space. How many degrees of freedom does it have? Propose a set of generalized coordinates.

A.: 5

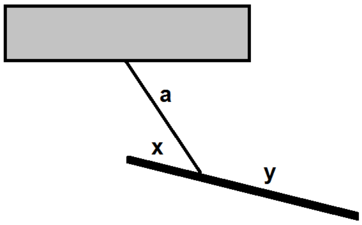
PROBLEM

A system consists of two rods which moves freely in a plane, but one end of the first rod is connected to the end of the second one by a rotatory joint. How many degrees of freedom does the system have? How many degrees of freedom does a chain of n rods moving in a plane have?

A.: for n rods f=n+2

PROBLEM

A thin rod hangs in string (see figure). It can move in three dimensional space. How many degrees of freedom does the system have?

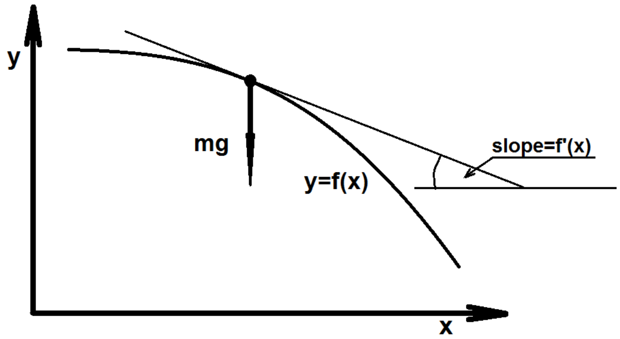


A.: 4

Generalized coordinates, Lagrange equations

PROBLEM 6

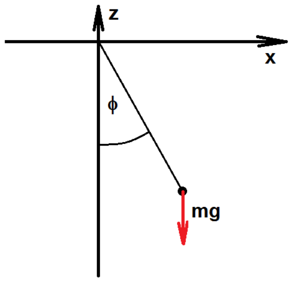
A point-like particle is moving under the influence of gravity at the curve y=f(x) on a vertical plane, y axis is vertical and x axis is horizontal. What is the generalized force when x is chosen as a generalized coordinate? What is physical meaning of the generalized force?



A.: $Q_x = -mgf'$. Q_x is not the tangential component of the gravity force.

PROBLEM 7

What is the generalized force in case of plane mathematical pendulum under the influence of gravity? The generalized coordinate id ϕ, the angle from the vertical.



A.: $Q_\phi = mgl\sin\phi$; torque of the gravity force

PROBLEM 9

Write Lagrange equations for a freely moving particle in spherical coordinates.

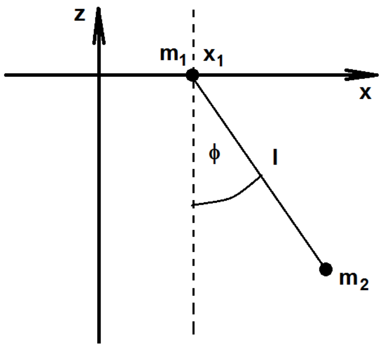
$$\ddot{r} - r\left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta\right) = 0$$

$$\frac{d}{dt}\left(r^2\dot{\phi}\sin^2\theta\right) = 0$$

$$\frac{d}{dt}\left(r^2\dot{\theta}\right) - r^2\dot{\phi}^2 \sin\theta\cos\theta = 0$$

PROBLEM 11

Find and discuss motion (in case of small oscillations) of a plane mathematical pendulum hanging on a point-like particle which can move freely at a horizontal straight line in the plane of motion of pendulum.



A.:

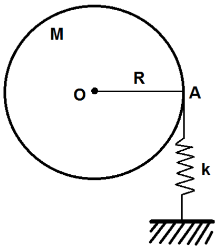
$$m_2 l\left(\ddot{x}_1 + l\ddot{\phi}\right) + m_2 g l\phi = 0$$

$$m_1\ddot{x}_1 + m_2\left(\ddot{x}_1 + l\ddot{\phi}\right) = 0$$

and finally $\ddot{\phi} + \frac{g}{l}\left(1 + \frac{m_2}{m_1}\right)\phi = 0$

PROBLEM 12

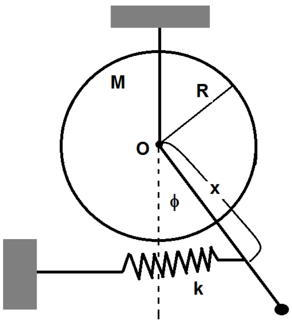
Using Lagrange equations find and discuss motion of a cylinder rotating round a fixed horizontal axis. Point A (see figure) of the cylinder is connected with a spring to a fixed point. The force constant of the spring is k. The spring is not stretched if the segment OA is horizontal.



A.: $\ddot{\phi}+\frac{kR^2}{I}\phi=0$

PROBLEM 13

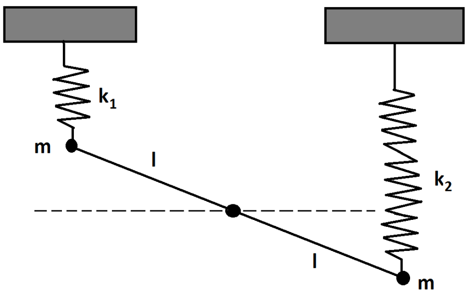
Find the plane motion of a system shown in the figure. Use the approximation of small oscillations. The axis O is fixed, the pendulum of length l is stiffly attached to the cylinder.



A.: $\ddot{\phi}+\frac{mgl+kx^2}{ml^2+I}\phi=0$

PROBLEM 14

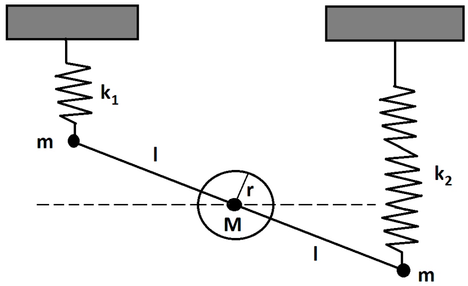
Find Lagrange's equation of a plane system shown in the figure and discuss its motion. Consider the case of small oscillations. The springs are not stretched when the massless rod is horizontal.



A.: $\ddot{\phi}+\frac{2(k_1+k_2)}{m_1+m_2}\phi=0$

PROBLEM 15

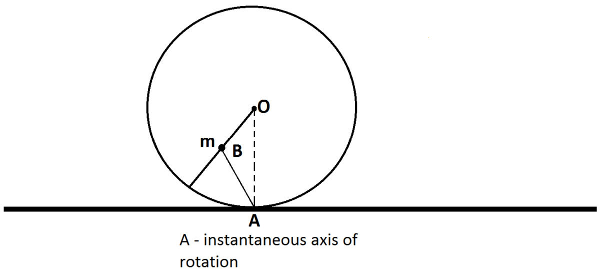
Find Lagrange's equation of a plane system shown in the figure. Consider the case of small oscillations. The springs are not stretched when the massless rod is horizontal.



A.: $\ddot{\phi} + \frac{2(k_1 + k_2)}{m_1 + m_2 + I}\phi = 0$

PROBLEM 16

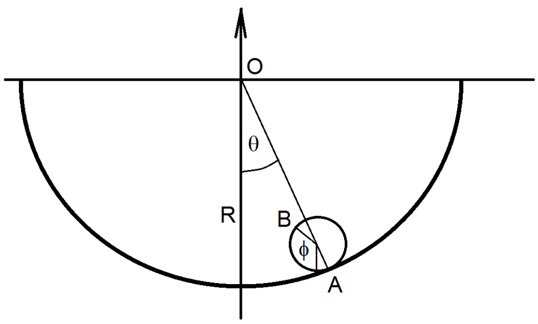
Find small oscillations of a cylindrical shell of radius R. Mass of the shell is M, the shell rolls on a horizontal plane without slipping. In the middle of shall radius point mass m is attached t a massles rod (see figure).



A.: $\ddot{\phi} + \frac{mgR}{2R^2\left(\frac{3}{2}M + \frac{1}{4}m\right)}\phi = 0$

PROBLEM 17

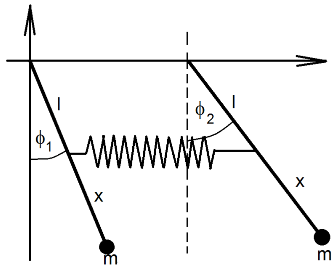
A homogeneous sphere of radius r rolls, without sliding, back and forth inside a cylindrical shell of radius R. Find Lagrange equations of a small plane oscillations and discuss the motion. Consider the case r<<R. What is the relation between the angles ϕ and θ for the case of comparable radii R and r.



A.: For r<<R $\left\{\frac{(R-r)^2}{R^2} + \frac{2}{5}\right\}\ddot{\phi} + \frac{g(R-r)}{R^2}\phi = 0$

PROBLEM 18

Find Lagrange equations of plane motion of two identical pendulums connected with an elastic spring (sympathetic pendulum). The spring is not stretched when the pendulums hang vertically. Consider the case of small oscillations.



A.: $m(l+x)^2\ddot{\phi}_1 + mg(l+x)\phi_1 - kl^2(\phi_2-\phi_1) = 0$

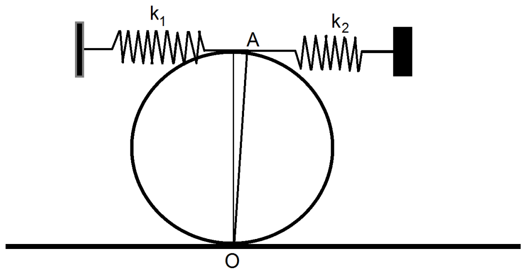
$m(l+x)^2\ddot{\phi}_2 + mg(l+x)\phi_2 + kl^2(\phi_2-\phi_1) = 0$

and finally $(l+x)(\ddot{\phi}_1+\ddot{\phi}_2) + g(\phi_1+\phi_2) = 0$

Sum of the two angles $\phi_1+\phi_2$ satisfies the equation of simple oscillations.

PROBLEM 19

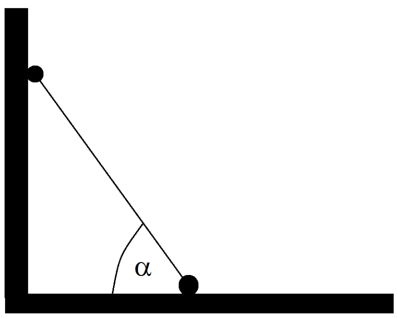
Homogeneous cylinder of radius r and mass m can roll without sliding on a horizontal plane. Two springs are connected to point A (see figure). Segment OA is vertical when the springs are not stretched.



A.: $\ddot{\phi} + \frac{16}{3}\frac{k}{m}\phi = 0$

PROBLEM 20

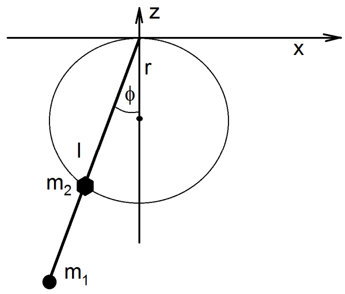
Two point-like particles of mass m are connected with a massless rod of length l. The rod is leant against a vertical wall as shown in the figure. There is no friction between the rod and the wall and the floor. Using Lagrange equations find equation of motion of the system.



A.: $\ddot{\alpha} + \frac{g}{l}\cos\alpha = 0$

PROBLEM 21

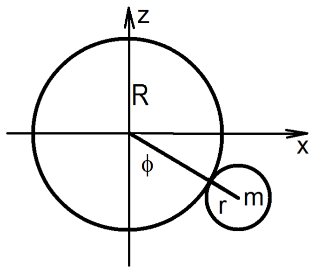
A point-like particle of mass m_1 is attached to one end of a massless rod of length l. The second end of the rod can rotate round fixed point O in the plane xz (see figure). A ring of mass m_2 can slide along the rod and a circle of radius r. Find motion of such a system in case of small oscillations.



A.: $\left(ml^2 + 4m_2r^2\right)\ddot{\phi} + \left(m_1gl + 4m_2gr\right)\phi = 0$

PROBLEM 22

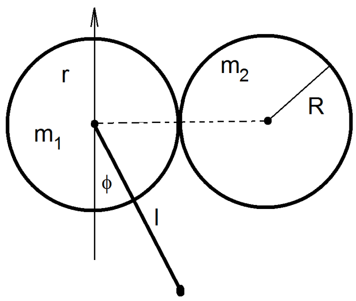
A sphere of mass m and radius r rolls on surface of cylinder of radius R as shown in the figure. The rod connecting the centre of sphere and cylinder of length l is massless. Find small oscillations of such a system. Consider the case r<<R. What is the relation between the angles ϕ and θ for the case of comparable radii R and r.



A.: $\left\{m(R+r)^2 + I\frac{R^2}{r^2}\right\}\ddot{\phi} + mg(R+r)\phi = 0$

PROBLEM 23

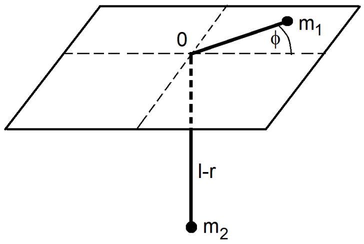
Two homogeneous cylinders can rotate without sliding round two parallel axis as shown in the figure. A pendulum is attached firmly to one of the cylinders. A restoring moment of force M=-kθ acts on the second cylinder. Find the equation of motion of such a system.



A.: $\ddot{\phi} + \dfrac{m_2gl + k\dfrac{r^2}{R^2}}{m_2l^2 + I_1 + I_2\dfrac{r^2}{R^2}}\phi = 0$

PROBLEM 24

Two point-like particle are connected with a rope (see figure). The rope passes through a hole in a smooth horizontal table. The first point can move horizontally on the table, the second one can move at a vertical straight line. Find the velocity of the second point as a function of the radius r.



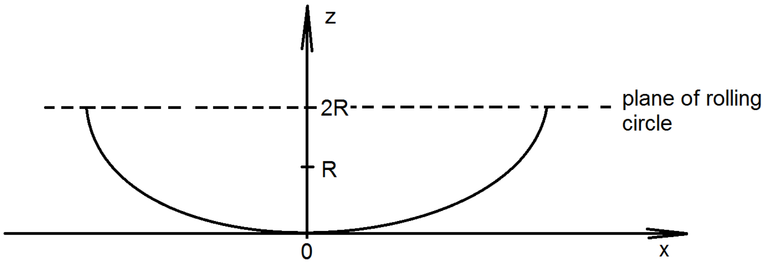
A.: $\dot{r}=\sqrt{\dfrac{2C_2m_1r^2-2m_1m_2gr^3-C_1^{\,2}}{m_1r^2(m_1+m_2)}}$; C_1 and C_2 are constants.

PROBLEM 25

Find plane motion of a point-like particle moving at a cycloid given by the equations

$x=R(\phi+\sin\phi)$

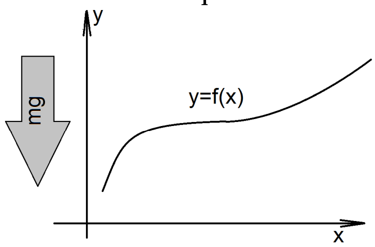
$z=R(1-\cos\phi)$



A.: Lagrange function $L=mR^2\dot{\phi}^2(1+\cos\phi)-mgR(1-\cos\phi)$. Substituting $1+\cos\phi=2\cos^2(\phi/2)$ and $1-\cos\phi=2\sin^2(\phi/2)$ we get $L=2mR^2\dot{\phi}^2\cos^2\frac{\phi}{2}+2mgR\sin^2\frac{\phi}{2}$. Substituting $s=4R\sin\frac{\phi}{2}$ (radius of curvature) we get $m\ddot{s}+\frac{mg}{4R}s=0$. Simple oscillations when parameter s is used to describe the motion.

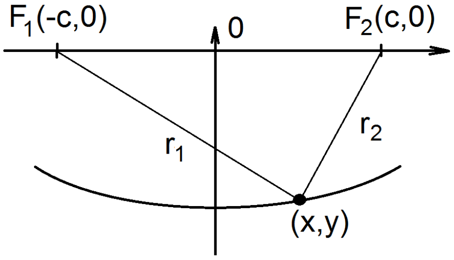
PROBLEM 26

Write Lagrange equations for a particle moving at a curve y=f(x), the y axis is vertical, in the presence of gravitation. Apply the obtained equation for a case of inclined plane.



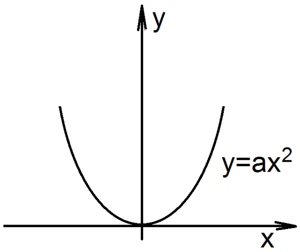
A.: $\ddot{x}(1+f'^2)+f'f''\dot{x}^2+gf'=0$

PROBLEM 27
A bead is sliding without friction along a massless string. The endpoints of the string are fixed at (x,y)=(-0.5s,0) and (0.5s,0) and the length of the string is $s\sqrt{2}$. Gravity acts along the negative z axis. Find the frequency of small oscillations of the bead around the stable equilibrium position.



A.: $\omega = \sqrt{\frac{g}{2s}}$

PROBLEM 28
A particle is sliding at a parabolic curve given by y=ax^2 in the presence of gravity. Find the frequency of a small oscillations. Axis y is vertical.

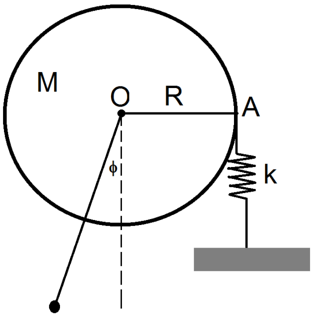


A.: $\omega = \sqrt{ag}$

PROBLEM 29
A particle moves at a curve y=f(x), the y axis is vertical. What is the angular frequency of small oscillations around a local minimum at x=x_0.

A.: $\omega = \sqrt{gf''(x_0)}$

PROBLEM 30
Using Lagrange equations find equations of motion of a cylinder rotating around a fixed horizontal axis and a mathematical pendulum attached to the cylinder as shown in the figure. Point A of the cylinder is connected with a spring to a fixed point. The force constant of the a spring is k. The spring is not stretched if the segment OA is horizontal.



PROBLEM 31

Using Lagrange equations solve the problem of oscillations of a system shown in the figure. The platform rolls on a horizontal plane without sliding.