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AN ANALYSIS OF THE THEORY OF TRANSFER LENGTH METHOD

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In this paper we present derivation of the equations describing the resistance between the electrodes used in The Transfer Length Method, both for the linear and circular configurations. For the circular configuration, we have obtained an approximated formula which is more accurate than the most widely used formula. Conditions allowing for verification if the approximations can be applied are also presented.

Keywords: Transfer Length Method, modified Bessel functions.

1. INTRODUCTION

Since the 1960s the Transmission Line Method (TLM) has been a basic method for deremi-nantion of the contact resistivity of metalic electrodes deposited on a semiconductor [1–4]. It allows also for determination of the sheet resistance of the semiconductor material on which the electrode is deposited. Usually, the measurement of the resistivity of the semiconductor is performed if the semiconductor layer is thin relative to the lateral dimensions of the electrodes because in this case the theory and hence the interpretation of the experimental data is much simpler than in a more general case [2].

TLM measurements are performed using a set of either rectangular electrodes (usually called simply a TLM measurement), or using electrodes in form of concentric rings (circular TLM or cTLM). In this paper, a detailed derivation of the formulas used for interpretation of the experimental data and a discussion of the important approximations often used in practical applications is presented.

The solutions for equations describing the current flow between the TLM electrodes are distinctly different in the case of linear and cylindrical configurations. First, a simpler case will be analysed.

2. LINEAR TLM

A linear TLM structure consists of a series of rectangular electrodes separated by a distance which is different for each to adjacent electrodes. In Figure 1 a top view of a part of such a structure is presented.

For each of the electrode pair, a current-voltage (I–V) curve is measured. Assuming that this curve is linear, the electrical resistance R for this pair can be determined. This resistance is a result of the resistance caused by the contact between the metal electrode and the semiconductor it is deposited on, and by the resistance of the semiconductor itself. The resistance of the metal can be neglected. In what follows, we will derive a formula for R. We will use the following approximation (see Figure 1 for the definitions of the symbols):

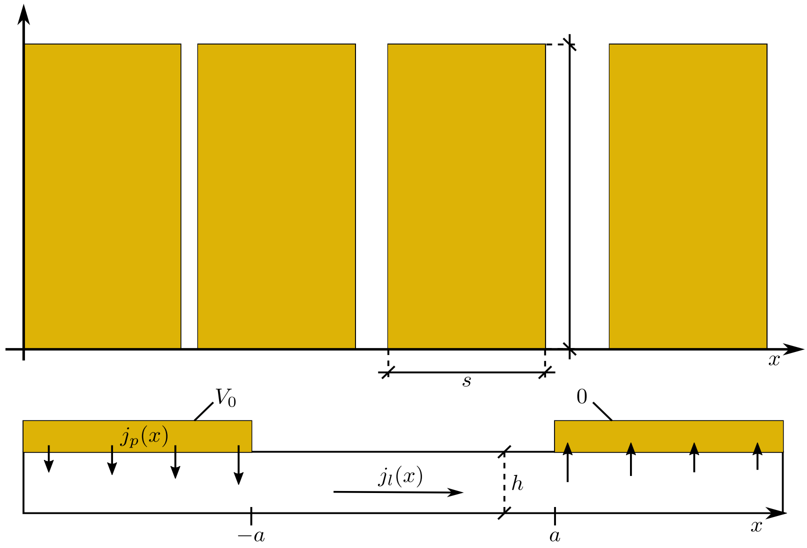

Figure 1. On the top: Top view of part of a linear TLM structure. On the bottom: a side cross-section of an area between two neighbouring electrodes
Source: own work.

- The thickness h of the semiconductor layer is very small, so that we can assume that current flows uniformly in the vertical direction and hence we can neglect this dimension in our calculations;
- The current density does not depend on y. This can be true if the distances between the electrodes are much smaller than their width, or if the TLM structure is fabricated in such a way that the current cannot spread beyond the area determined by the electrodes' width;
- The electrodes are semi-infinite in the x direction (i.e. in Figure 1 $s = \infty$); the potential of the left electrode if V_0, and 0 on the right electrode. Later, we will discuss the case in which $s < \infty$.

The density of the vertical current which flows from the electrode into the semiconductor (or the other way round) depends on x and can be expressed as:

$$j_p(x) = \varsigma_c\big(V_c - u(x)\big) \tag{1}$$

where $u(x)$ is the local electric potential; V_c is the electrode potential (equal to either V_0 or 0); and ς_c is the surface conductivity of the electrode–semiconductor interface (the unit of ς_c is $\mathrm{S/m^2}$). The above formula is valid if x refers to a point covered by the electrode, otherwise $j_p = 0$. The horizontal current I_l and its density j_l for any x are given by the following formula:

$$j_l(x) = -\sigma_s u'(x) \tag{2}$$

$$I_l(x) = hwj_l(x) \tag{3}$$

where σ_s is the electrical conductivity of the semiconductor material. On the other hand, the horizontal current in the area below an electrode (i.e. for $|x| > a$) is related to j_p in the following manner:

$$I_l(x) = \begin{cases} w \displaystyle\int_{-\infty}^{x} j_p(\xi)\, d\xi & \text{for } x < -a \\ I_l(a) + w \displaystyle\int_{a}^{x} j_p(\xi)\, d\xi & \text{for } x > a \end{cases} \tag{4}$$

We differentiate both sides of the above equation with respect to x, substitute (3) and we get the following equation (valid both for $x > a$ and $x < -a$):

$$hwj_l'(x) = wj_p(x) \tag{5}$$

By reducing w and substituting (1) and (2) we get:

$$u''(x) - \gamma^2 u(x) + \gamma^2 V_c = 0 \tag{6}$$

where

$$\gamma^2 = \frac{\varsigma_c}{h\sigma_s} \tag{7}$$

We could assume that the conductivities of the left and the right electrode are different ($\varsigma_c^{(1)}$ and $\varsigma_c^{(2)}$ respectively). Then we would have coefficients γ_1 for $x < -a$ and γ_2 for $x > a$. The general solutions in these two areas are as follows:

$$u_1(x) = C_1 \exp(\gamma_1 x) + \tilde{C}_1 \exp(-\gamma_1 x) + V_0 \qquad \text{for } x < -a \tag{8}$$

$$u_2(x) = \tilde{C}_2 \exp(\gamma_2 x) + C_2 \exp(-\gamma_2 x) \qquad \text{for } x > a \tag{9}$$

For $x \to -\infty$ potential u has to tend to V_0, while for $x \to \infty$ to 0 (which are the potential of the respective electrodes). Otherwise, an infinite current would flow through the structure. It means that both coefficients $\tilde{C}$ must vanish and hence:

$$u_1(x) = C_1 \exp(\gamma_1 x) + V_0 \qquad \text{for } x < -a \tag{10}$$

$$u_2(x) = C_2 \exp(-\gamma_2 x) \qquad \text{for } x > a \tag{11}$$

Between the electrodes (for $|x| < a$), current $I_l(x)$ and hence its density $j_l(x)$ must be constant and equal, respectively, to I (which is the total current flowing between the electrodes) and j. Using Eq. (2) we conclude that in this area $u''(x) = 0$ which means that u is a linear function:

$$u(x) = -\frac{j}{\sigma_s}\, x + b \quad \text{for } |x| < a \tag{12}$$

Continuity of $j_l(x)$ end Eqs. (2), (10) and (11) give us the following relation:

$$-C_1 \sigma_s \gamma_1 \exp(-\gamma_1 a) = j = C_2 \sigma_s \gamma_2 \exp(-\gamma_2 a) \tag{13}$$

and hence:

$$\frac{C_1}{C_2} = -\frac{\gamma_2}{\gamma_1} \exp\big(-(\gamma_2 - \gamma_1)a\big) \tag{14}$$

Continuity of $u(x)$ at $x = -a$ and $x = a$ provides further two equations which can be used to determine parameters C_1, C_2, b, j. The parameter we really need is j, since it gives us the total current I. After some elementary calculations we get:

$$\frac{V_0}{j} = \frac{1}{\sigma_s}\left(2a + \gamma_1^{-1} + \gamma_2^{-1}\right) \tag{15}$$

Let $L = 2a$ denote the distance between the electrodes. The resistance between them is equal to V_0/I, so after substituting (7) we obtain:

$$R(L) = \frac{L}{hw\sigma_s} + \frac{1}{w\sqrt{h\sigma_s}}\left(\frac{1}{\sqrt{\varsigma_{c1}}} + \frac{1}{\sqrt{\varsigma_{c2}}}\right) = \frac{L}{hw\sigma_s} + \frac{2\sqrt{\varrho_c}}{w\sqrt{h\sigma_s}} \tag{16}$$

where

$$2\sqrt{\varrho_c} = \frac{1}{\sqrt{\varsigma_{c1}}} + \frac{1}{\sqrt{\varsigma_{c2}}} \tag{17}$$

Since Eq. (15) γ_1^{-1} and γ_2^{-1} are simply added, in the linear TLM experiment it is impossible to determine separately the resistivities of the individual electrodes. Instead, we can use a single γ. However, in principle these resistivities should be the same, so this is not considered a problem.

In the TLM experiment $R(L)$ is measured for several values of L. This relation should be linear, as predicted by Eq. (16), and the experimental data provide parameters α and R_0 in the following relation:

$$R(L) = \alpha L + R_0 \tag{18}$$

Combining the above experimental parameters and the theoretical relation we obtain two relations for the semiconductor electrical conductivity (in the horizontal direction) and the contact resistivity:

$$\sigma_s = \frac{1}{hw\alpha} \tag{19a}$$

$$\varrho_c = \frac{1}{4}R_0^2 w^2 h\sigma_s = \frac{wR_0^2}{4\alpha} \tag{19b}$$

The simple and practical formula (19) was derived under the assumption that the electrodes can be considered semi-infinite along x direction. Now we will provide a criterion to verify if this assumption is valid in individual cases.

2.1. Verification if the semi-infinite electrodes assumption is valid

The relations derived so far assume that the current flowing from an electrode to the semiconductor uses the whole infinite length of the electrode, while in fact the available length is equal to s. Let us calculate the current which flows through the part of an electrode which in fact does not exist. Integrating formula (1) with the proper limits and using Eq. (13) and (2) we obtain the following formulas for the total current I and the current injected to the actually existing part of the second electrode (equal to the current injected by the actually existing part of the first electrode):

$$I = \int_a^\infty \varsigma_c u(x) = \frac{wj}{\sigma_s\gamma^2} \tag{20}$$

$$I_s = \int_a^{a+s} \varsigma_c u(x) = \frac{wj}{\sigma_s\gamma^2}(1 - \exp(-\gamma s)) \tag{21}$$

If the difference between those two currents is negligibly small (relative to the total current), the simplification we used can be considered valid. This gives the following criterion which can be used for such a validation:

$$\exp(-\gamma s) \ll 1 \iff \exp\left(-\frac{s}{\sqrt{h\varrho_c\sigma_c}}\right) = \exp\left(-\frac{2s\alpha}{R_0}\right) \ll 1 \tag{22}$$

What if one cannot apply the considered simplification?

2.2. Solution for finite electrodes

The problem with finite electrodes can be solved. The differential equations derived above are still valid. What is different is the boundary conditions. Instead of

$$\lim_{x\to\pm\infty} j_l(x) = 0 \tag{23}$$

now we have to use

$$\lim_{x\to\pm(a+s)} j_l(x) = 0 \tag{24}$$

In this case, we will use a more convenient form of the general solution (6)

$$u_1(x) = A_1 \cosh\big(\gamma_1(x-x_1)\big) + \tilde{A}_1 \sinh\big(-\gamma_1(x-x_1)\big) + V_0 \qquad \text{for } x < -a \tag{25}$$

$$u_2(x) = \tilde{A}_2 \cosh\big(\gamma_2(x-x_2)\big) + A_2 \sinh\big(-\gamma_2(x-x_2)\big) \qquad \text{for } x > a \tag{26}$$

where $x_1 = -a - s$ and $x_2 = a + s$ are the outer boundaries of the electrodes. Then, using (24) one can see that $\tilde{A}_1 = \tilde{A}_2 = 0$. The properties of the hiperbolic functions allow us to write (13) in the following way:

$$-A_1\sigma_s\gamma_1 \sinh(\gamma_1 s) = j = A_2\sigma_s\gamma_2 \sinh(\gamma_2 s) \tag{27}$$

Relation (12) remains true and combining it with the above formula we obtain a more general version of Eq. (15):

$$\frac{V_0}{j} = \frac{1}{\sigma_s}\left(2a + \gamma_1^{-1}\coth(\gamma_1 s) + \gamma_2^{-1}\coth(\gamma_2 s)\right) \tag{28}$$

Because $\coth(x) \xrightarrow[x\to\infty]{} 1$, under condition (22) the above formula reduces to the form presented in Eq. (15).

The exact solution does not change the formula for σ_s, however ϱ_c is given in an implicit form. The non-simplified version of relations (19) can be expressed in the following form, assuming that $\gamma_1 = \gamma_2 = \gamma$:

$$\sigma_s = \frac{1}{hw\alpha} \tag{29a}$$

$$R_0 = \frac{2\coth(\gamma s)}{hw\sigma_s\gamma} = 2\sqrt{\frac{\alpha\varrho_c}{w}}\coth\left(s\sqrt{\frac{w\alpha}{\varrho_c}}\right) \tag{29b}$$

In the analysis of linear TLM presented above, we always assumed that the current flow is bounded to the area of the width equal to the electrodes' width. This is a sound assumption only if the distance between the electrodes is much smaller than the electrode width. However, if the conductivity of the semiconductor material is high, the resistance between the electrodes can be vary small relative to the contact resistance. This can severely deteriorate the accuracy of the determination of the semiconductor conductivity. In such a case, a cTLM measurement may be a better choice.

3. CIRCULAR TLM

A single cTLM electrode pair has a form of two concentric rings, where the inner ring is usually a full disk, as shown in Figure 2. In a cTLM experiment the resitivities for several such pairs, with different dimensions, are measured. In this section, we will derive a theoretical formula for the resistivity of such a pair. In our derivation we will use polar co-ordinates. We will also use the first of the three approximations used in section 2.

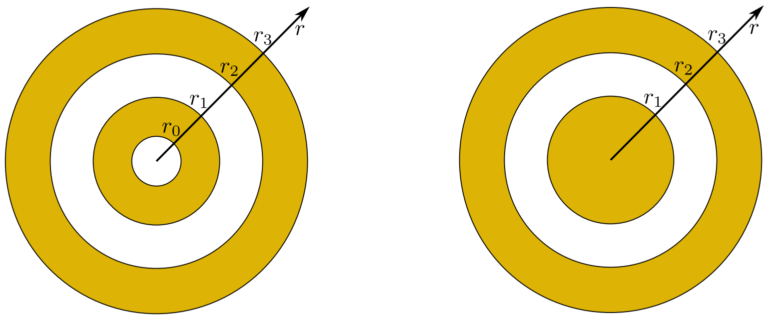

Figure 2. A top view of two posible configurations of an electrode pair used in the cTLM experiment

Source: own work.

The density of the (horizontal) current flowing between the electrodes through the semiconductor has only the radial component j_l, which is independent from the angle, because of the rotational symmetry of the system. Similarly, the electrical potential u depends only on the radial co-ordinate r. In polar co-ordinates, the analogues to formulas (1) and (2) have the following form:

$$j_p(r) = \varsigma_c\big(V_c - u(r)\big) \tag{30a}$$

$$j_l(r) = -\sigma_s u'(r) \tag{30b}$$

where the symbols have the same meaning as in section 2. We will consider a circular sector with its central angle φ. For $\varphi = 2\pi$ we will get the whole plane. The formula that describes the relation between the current flowing through this sector and its density looks as follows:

$$I_l(r) = hr\varphi j_l(r) \tag{31}$$

In the area beneath the electrodes we additionally have the following relations:

$$I_l(r) = \begin{cases} \varphi \displaystyle\int_{r_0}^{r} \xi j_p(\xi)\, d\xi & \text{for } r_0 < r < r_1 \\ I_l(r_2) + \varphi \displaystyle\int_{r_2}^{r} \xi j_p(\xi)\, d\xi & \text{for } r_2 < r < r_3 \end{cases} \tag{32}$$

By differentiating formulas (31) and (32), substituting Eq. (30) and (7) we obtain the following equation (valid below the electrodes):

$$r^2 u''(r) + ru'(r) - \gamma^2 r^2\big(u(r) - V_c\big) = 0 \tag{33}$$

The general solution for this equation has the following form

$$u(r) = B_I I_0(\gamma r) + B_K K_0(\gamma r) + V_c \tag{34}$$

where I_0 i K_0 are the modified Bessel functions. The equation for the potential between the electrodes is much simpler:

$$u'(r) + ru''(r) = 0 \tag{35}$$

Its solution, for $\varphi = 2\pi$ can be expressed using the total current I in the following way:

$$u_m(r) = -\frac{I}{2\pi h\sigma_s} \ln\left(\frac{r}{p}\right) \tag{36}$$

where p is a parameter whose value we are yet to determine.

In what follows we will simplify our considerations assuming that $r_0 = 0$ (which is how the inner electrode is usually fabricated) and that $r_3 \to \infty$. The latter condition is very similar to the third approximation used in section 2. Then, using the following properties of the Bessel function:

$$\lim_{\xi\to 0} K_0(\xi) = \lim_{\xi\to\infty} I_0(\xi) = \infty \tag{37}$$

$$\lim_{\xi\to 0} I_0(\xi) = \lim_{\xi\to\infty} K_0(\xi) = 0 \tag{38}$$

we obtain

$$u_1(r) = B_1 I_0(\gamma_1 r) + V_1 \tag{39a}$$

$$u_2(r) = B_2 K_0(\gamma_2 r) + V_2 \tag{39b}$$

where u_1, u_2 are the electrical potentials in the semiconductor below, respectively, the inner and the outer electrode, while V_1, V_2 are the respective electrode potentials. Continuity of the current at r_1 and r_2 provides the following equations:

$$-h\sigma_s B_1\gamma_1 2\pi r_1 I_0'(\gamma_1 r_1) = I = -h\sigma_s B_2\gamma_2 2\pi r_2 K_0'(\gamma_2 r_2) \tag{40}$$

Using the following properties of the modified Bessel function [5]:

$$I_n' = \frac{1}{2}(I_{n-1} + I_{n+1}) \tag{41a}$$

$$K_n' = -\frac{1}{2}(K_{n-1} + K_{n+1}) \tag{41b}$$

$$I_{-1} = I_1 \tag{41c}$$

$$K_{-1} = K_1 \tag{41d}$$

we obtain the following formula:

$$-\sigma_s B_1\gamma_1 2\pi r_1 I_1(\gamma_1 r_1) = I = \sigma_s B_2\gamma_2 2\pi r_2 K_1(\gamma_2 r_2) \tag{42}$$

The resistance between the electrodes $(V_1 - V_2)/I$ can be expressed, using Eqs. (36), (39) and (40) in the following way:

$$R(r_1, r_2) = \frac{1}{2\pi h\sigma_s}\left(\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{r_1\gamma_1}\frac{I_0(r_1\gamma_1)}{I_0'(r_1\gamma_1)} - \frac{1}{r_2\gamma_2}\frac{K_0(r_2\gamma_2)}{K_0'(r_2\gamma_2)}\right) \tag{43}$$

or, using Eq. (41):

$$R(r_1, r_2) = \frac{1}{2\pi h\sigma_s}\left(\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{r_1\gamma_1}\frac{I_0(r_1\gamma_1)}{I_1(r_1\gamma_1)} + \frac{1}{r_2\gamma_2}\frac{K_0(r_2\gamma_2)}{K_1(r_2\gamma_2)}\right) \tag{44}$$

Neither of the two relations is very convenient. Let us try to get rid of the modified Bessel functions using an approximation which is valid if $|x|$ is sufficiently large [6]:

$$I_0(x) \approx \frac{e^x}{\sqrt{2\pi x}} \tag{45}$$

$$K_0(x) \approx \sqrt{\frac{\pi}{2\pi x}} e^{-x} \tag{46}$$

It allows us to find a convenient estimation of the Bessel function to their derivatives:

$$\frac{I_0(x)}{I_0'(x)} \approx \frac{2x}{2x-1} \tag{47}$$
$$\frac{K_0(x)}{K_0'(x)} \approx -\frac{2x}{2x+1} \tag{48}$$

In the literature, often the above two ratios is approximated simply by 1 [7, 8]. Below we present three formulas for the resistance, starting from the most accurate and ending on the least accurate of them:

$$R(r_1, r_2) = \frac{1}{2\pi h\sigma_s}\left(\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{r_1\gamma_1}\frac{I_0(r_1\gamma_1)}{I_1(r_1\gamma_1)} + \frac{1}{r_2\gamma_2}\frac{K_0(r_2\gamma_2)}{K_1(r_2\gamma_2)}\right) \tag{49}$$
$$\approx \frac{1}{2\pi h\sigma_s}\left(\ln\left(\frac{r_2}{r_1}\right) + \frac{2}{2r_1\gamma_1 - 1} + \frac{2}{2r_2\gamma_2 + 1}\right) \tag{50}$$
$$\approx \frac{1}{2\pi h\sigma_s}\left(\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{r_1\gamma_1} + \frac{1}{r_2\gamma_2}\right) \tag{51}$$

Unlike in the linear TLM case, in the above formulas the contributions from the individual electrodes (described by γ_1 and γ_2) is not the same. If, hypothetically, the electrode resistivity depended on the direction of the current flow (but otherwise were perfectly linear), a measurement of the IV curve for a cTLM structure, would show a difference in slope between the negative and positive current side. If this in not the case, we can say that $\gamma_1 = \gamma_2 = \gamma$, and the above formulas can be written in the following form:

$$R(r_1, r_2) = \frac{1}{2\pi h\sigma_s}\left(\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{r_1\gamma}\frac{I_0(r_1\gamma)}{I_1(r_1\gamma)} + \frac{1}{r_2\gamma}\frac{K_0(r_2\gamma)}{K_1(r_2\gamma)}\right) \tag{52}$$
$$\approx \frac{1}{2\pi h\sigma_s}\left(\ln\left(\frac{r_2}{r_1}\right) + \frac{2}{2r_1\gamma - 1} + \frac{2}{2r_2\gamma + 1}\right) \tag{53}$$
$$\approx \frac{1}{2\pi h\sigma_s}\left(\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{\gamma}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)\right) \tag{54}$$

3.1. Validity of the Bessel functions approximations

Let us analyze when the approximations considered above are valid. The exact solution Eq. (49) (however assuming that the outer ring is semi-infinite) contains terms in the form $B(x)/B'(x)$ where B is a modified Bessel function: either I_0 or K_0. In the approximation (50) and (51) we used the following sequence of simplifications:

$$\frac{I_0(x)}{I_0'(x)} = \frac{I_0(x)}{I_1(x)} \approx \frac{2x}{2x-1} \approx 1 \tag{55}$$
$$-\frac{K_0(x)}{K_0'(x)} = \frac{K_0(x)}{K_1(x)} \approx \frac{2x}{2x+1} \approx 1 \tag{56}$$

The accuracy for both approximations improves with $|x| \to \infty$. However, we do not know yet when x is sufficiently large. Now, we will try to answer this question. Let us denote:

$$
\begin{aligned}
Q_I(x) &= \frac{I_0(x)}{I_1(x)} & q_I(x) &= \frac{2x}{2x-1} & \tilde{q}(x) &= 1 \\
Q_K(x) &= \frac{K_0(x)}{K_1(x)} & q_K(x) &= \frac{2x}{2x+1} \\
r_I(x) &= 1 - \frac{q_I(x)}{Q_I(x)} & \tilde{r}_I(x) &= 1 - \frac{\tilde{q}(x)}{Q_I(x)} \\
r_K(x) &= 1 - \frac{q_K(x)}{Q_K(x)} & \tilde{r}_K(x) &= 1 - \frac{\tilde{q}(x)}{Q_K(x)}
\end{aligned}
$$

Functions r and $\tilde{r}$ are simply the relative error of approximation q and $\tilde{q}$ respectively. Figure 3 presents a comparison of the approximations used in this paper. If we assume that an error below 1% is acceptable, we can see that approximation q, and hence formula (50) is valid if $\gamma r_1 > 4.5$, while formula (51) should not be used be used unless $\gamma r_1 > 50$. Since formula (50) is only slightly more complicated than formula (51), there is probably no reason to use the latter at all for fitting to experimental data. However, it seems that Eq. (50) does not appear in the literature and only Eqs. (49) and (51) are reported.

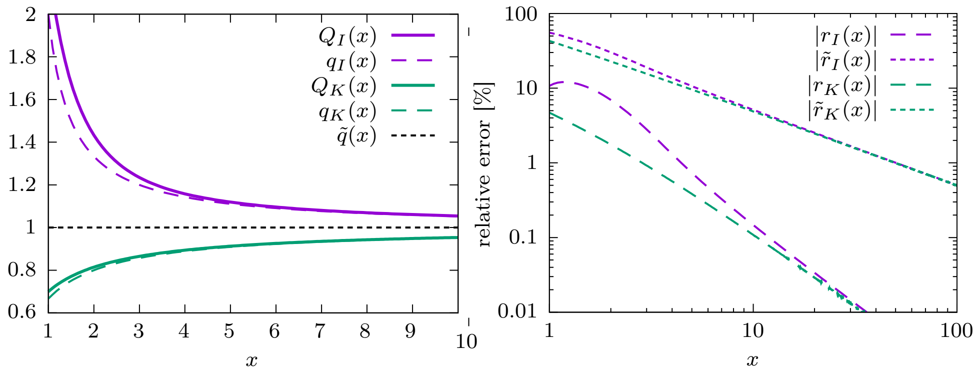

Figure 3. Left: Comparison of the considered approximations of the ratios of the Bessel functions; Right: The relative errors of the approximations
Source: own work.

The above estimations are important not only to validate if the formula used for determination of parameters γ and σ_s was correct. If Eq. (50) is to be used, then one has to accept that for the radii of the outer rings greater than $\sim 1.4r_0$ the combined resistance of both electrodes is less than 10% of the total resistance. This greatly (and negatively) impacts the accuracy of determination of the contact resistance in such an experiment.
Of course, whether or not the contact resistance can be reliably determined in such an experiment depends on the system geometry and the material parameters. If $\gamma r_0 > 50$, the cTLM configuration itself is not ideal to determine the contact resistance. On the other hand, when the material conductivity is high, that it is hard to measure in the linear TLM configuration, cTLM can be a suitable method. Whatever the situation, since Eq. (51) compared with Eq. (50) does not seem to provide any significant simplification, there is probably no reason to use Eq. (51).

4. SUMMARY

No we will summarize the paper presenting the procedure of experimental data analysis. We assume that a set of I-V curves have been measured—one curve for each pair of electrodes, and provided that they are linear, for each of them the resistance was determined. If the I-V curves are not linear, then the electrode deposition was faulty, and the TLM theory presented here cannot be applied. If we want to determine σ_s we also need to know the thickness of the semiconductor layer.

4.1. Linear TLM

In the case of linear TLM, the experimental data give us a relation $R(L)$, where R is the resistance between electrodes separated by a distance L. A linear function $f(L) = \alpha L + R_0$ is fitted to the experimental data. Then, the condition given in Eq. (22) should be verified. If this condition is fulfilled, formulas (19) can be used to determine the investigated material parameters. If not, formulas (29) should be used, however in order to find ϱ_c a numerical root-finding will be necessary.

4.2. Circular TLM

When cTLM is used, we measure a function $R(r_1, r_2)$. Often, r_1 is identical in all the measured electrode pairs, and the we can assume that we have a function $R(r_2)$. Then we fit relation (53) to the experimental data, obtaining the values for σ_s and γ. If $r_1\gamma > 4.5$, formula (53) can be considered valid. If not, we have to perform least-square fitting again, using this time Eq. (52). Using Eq. (54) should be avoided.

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