Lodz University of Technology
Institute of Physics

Laboratory of electronics

Exercise E05IFE

Passive filters

Version 2.3 (7 April 2024)

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Before you start to perform an experiment you are obliged to have mastered the following theoretical subjects:

1. DC and AC currents. Voltage, current intensity and electric resistance. [1], [2].
2. Principle of electrical measurements. [3], [8], [9].
3. Four-terminal network. [1], [2].
4. Principle of oscilloscope operation. [4].
5. Passive electronic elements. [5], [6].
6. Electric circuits RC, LC and RLC. [6]-[9].

1. Purpose of the exercise

The purpose of the exercise is to obtain and analyze amplitude-frequency and phase-frequency characteristics of the following passive filters:
1) low-pass RC and LC filters,
2) high-pass RC and LC filters,
3) band-pass Wien RC and LC filters.

2. Hazards

Type	Absence	Low	Medium	High
electrical shock hazards		+		
optical radiation hazards	+			
mechanical hazards (including acoustic hazards, noise)	+			
electromagnetic radiation hazards (invisible)	+			
biological hazards	+			
ionizing radiation hazards	+			
chemical hazards	+			
thermal hazards (including explosion and fire)	+			

The cables with banana plugs are designed exclusively for use in low-voltage circuits – do not connect them to the mains supply 230 V.

3. Introduction

3.1. R, L, C elements in AC circuits

Each vector on the complex plane can be represented as a complex number. Regardless of the physical meaning of a given complex quantity, we will use the underlined symbols, e.g.:

$$\underline{A} = \mathrm{Re}(\underline{A}) + j\,\mathrm{Im}(\underline{A}) = A\exp(j\alpha) = A(\cos\alpha + j\sin\alpha)\,, \qquad (1)$$

where A is called the modulus or magnitude of a complex number $\underline{A}$, $j=\sqrt{-1}$ is the imaginary unit, $\mathrm{Re}(\underline{A})$ and $\mathrm{Im}(\underline{A})$ are the projections of the $\underline{A}$ vector onto the real and imaginary exes, respectively, and α is the argument of a complex number.

In sinusoidal AC circuits the phase difference between the current and voltage waveforms may occur. The relationships for such circuits can be easily written in the form of Ohm's law for complex quantities

$$\underline{U} = \underline{Z}\,\underline{I} \quad \text{or} \quad \underline{I} = \underline{Y}\,\underline{U}\,, \tag{2}$$

where $\underline{U}$ and $\underline{I}$ are the complex voltage and current, respectively, and $\underline{Z}$ is the complex impedance which has the dimension of electrical resistance [Ω], and $\underline{Y}$ is the complex admittance which has the dimension of electrical conductance [S] (siemens). The complex impedance may be written in the following forms

$$\underline{Z} = R + j\,X \quad \text{or} \quad \underline{Z} = Z\,\exp(j\phi)\,, \tag{3}$$

where: the magnitude of complex impedance $Z = \sqrt{R^2 + X^2}$ is called the impedance,

$\phi = \mathrm{tg}(X/R)$ is the phase difference between voltage and current,

$R = \mathrm{Re}(\underline{Z}) = Z\cos\phi$ is the resistance,

$X = \mathrm{Im}(\underline{Z}) = Z\sin\phi$ is the reactance (does not dissipate power).

Analogously the complex admittance

$$\underline{Y} = G + j\,B \quad \text{or} \quad \underline{Y} = Y\,\exp(j\phi')\,, \tag{4}$$

where: the magnitude $Y = \sqrt{G^2 + B^2}$ is called the admittance,

$\phi' = \mathrm{tg}(B/G)$,

$G = \mathrm{Re}(\underline{Y}) = Y\cos\phi'$ is the conductance,

$B = \mathrm{Im}(\underline{Y}) = Y\sin\phi'$ is the susceptance.

There are the following relationships between the impedance and the admittance:

$$\underline{Z} = \frac{1}{\underline{Y}} = \frac{G - jB}{Y^2}\,, \qquad \phi = -\phi'\,. \tag{5}$$

If a sinusoidal altering voltage with the frequency f is applied to the ideal coil with the inductance L [H], the voltage to phase difference $\phi = \pi/2$ is positive and the coil reactance is given by

$$X_L = 2\pi f L\,. \tag{6}$$

In the case of an ideal capacitor with a capacity of C [F], the reactance is

$$X_C = -\frac{1}{2\pi f C}\,, \tag{7}$$

where the negative sign is usually assumed due to the negative value of the phase difference $\phi = -\pi/2$.

The reactance of an LC series circuit can be calculated as the sum of the coil's reactance and the capacitor's reactance $X = X_L + X_C$. In the general case, the total complex impedance of the serial circuit composed of n impedances is given by

$$\underline{Z} = \underline{Z}_1 + \underline{Z}_2 + \ldots + \underline{Z}_n\,. \tag{8}$$

Admittance is particularly useful in analyzing parallel circuits. The total complex admittance of parallel circuit is the sum of the individual admittances

$$\underline{Y} = \underline{Y}_1 + \underline{Y}_2 + \ldots + \underline{Y}_n\,. \tag{9}$$

3.2. Voltage divider in AC circuits

All circuits tested in this exercise have the structure of voltage divider shown in Fig. 1, where, in the general case, the impedances of the components are described by the complex values $\underline{Z}_1$ and $\underline{Z}_2$.

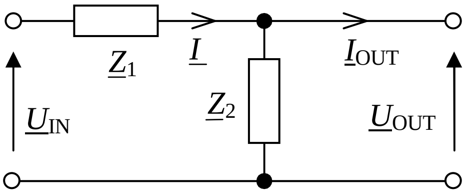

Fig. 1. Circuit diagram for a voltage divider.

If the current I_{OUT} flowing to the output of the divider is negligibly small, the same current I flows through the $\underline{Z}_1$ and $\underline{Z}_2$ impedances. The Ohm's law (2) written for the $\underline{Z}_2$ impedance and for the serial circuit composed of the $\underline{Z}_1$ and $\underline{Z}_2$ impedances takes the form

$$\underline{U}_{OUT} = \underline{Z}_2 \, \underline{I} \,, \tag{10}$$

$$\underline{U}_{IN} = (\underline{Z}_1 + \underline{Z}_2) \, \underline{I} \,. \tag{11}$$

Hence, by eliminating the current $\underline{I}$, we get

$$\frac{\underline{U}_{OUT}}{\underline{U}_{IN}} = \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} \,. \tag{12}$$

Since the equation (12) involves complex quantities, it will be convenient to analyze separately its real part describing the ratio of amplitudes or RMS (root mean square) voltages that can be easily measured

$$\frac{U_{OUT}}{U_{IN}} = \left| \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} \right| . \tag{13}$$

If the sinusoidal altering complex voltages are expressed in the form

$$\underline{U}_{IN} = U_{IN} \exp[j(\omega t + \varphi_{IN})] \quad \text{and} \quad \underline{U}_{OUT} = U_{OUT} \exp[j(\omega t + \varphi_{OUT})] \,, \tag{14}$$

the phase shift measured between the output and the input of the divider $\varphi = \varphi_{OUT} - \varphi_{IN}$ can be calculated as

$$\varphi = \mathrm{arctg} \frac{\mathrm{Im}\left(\underline{U}_{OUT} / \underline{U}_{IN}\right)}{\mathrm{Re}\left(\underline{U}_{OUT} / \underline{U}_{IN}\right)} \,. \tag{15}$$

In the case of LC filters the value of φ may go beyond the range $-\pi/2 \div +\pi/2$ covered by the arctangent function and the following formula should be used

$$\varphi = \frac{\mathrm{Im}\left(\underline{U}_{OUT} / \underline{U}_{IN}\right)}{\left|\mathrm{Im}\left(\underline{U}_{OUT} / \underline{U}_{IN}\right)\right|} \arccos \frac{\mathrm{Re}\left(\underline{U}_{OUT} / \underline{U}_{IN}\right)}{\left|\underline{U}_{OUT} / \underline{U}_{IN}\right|} \,. \tag{16}$$

3.3. Signal filters

Signal filter is a circuit with a four-terminal structure (one pair of terminals acts as an input and the other as an output) that passes signals in a specific frequency band, and attenuates signals with frequencies lying outside the band. Signal filters are mainly used in electronic and energy devices. The filters connected between the signal source and the receiver provide the signal with the desired frequency spectrum, which means that unwanted signals have been filtered out.

The frequency band in which the filter passes signals with low attenuation is called the pass-band and the band in which the signals are strongly attenuated is called the stop-band. The frequency at a boundary between these bands is the cut-off frequency. Some filters may have several cut-off frequencies. Filters may be classified according to range of signal frequencies in the pass-band:

- low-pass filters – pass-band from the frequency $f = 0$ Hz to the cut-off frequency f_c,
- high-pass filters – pass-band from the cut-off frequency f_c to infinity,
- band-pass filters – pass-band in the frequency range from the lower cut-off frequency f_{c1} to the upper cut-off frequency f_{c2},
- band-stop or band-reject filters – stop-band in the frequency range from f_{c1} to f_{c2}.

Depending on the components used, filters can be classified as:

- passive filters – contain only passive components:
  – non-inductive filters (RC) – consist only of resistors and capacitors,
  – reactance filters (LC) – consist only of coils and capacitors,
- active filters – the filter circuit contains active components, such as operational amplifiers. It allows one to design a filter with any frequency response.

The basic parameters characterizing any passive signal filter are:

1) **attenuation (*k*)** – a value describing what part of the input signal is passed to the filter output at a given frequency. It can be expressed in several ways, e.g. as a direct voltage ratio U_{OUT}/U_{IN} or attenuation measured in decibels

$$k = -20\log\frac{U_{OUT}}{U_{IN}} \text{ [dB]}, \tag{17}$$

2) **phase shift φ** – the difference between the phase of the voltage at the filter output and the phase of the voltage at the input expressed in degrees or radians,

3) **cut-off frequency (f_c)** – a frequency characterizing a boundary between a pass-band and a stop-band. Typically, this frequency is assumed to be the frequency at which the attenuation increases by 3 dB in comparison to the minimum attenuation in the pass-band (so-called "a 3 decibel cut-off frequency"). According to the formula (17), an increase in the attenuation by 3 dB corresponds to a decrease in the value of the U_{OUT}/U_{IN} ratio to the level of $10^{-3/20} \approx 0.708$ the maximum value in the pass-band. The 3 dB cut-off frequency is often equated with the cut-off frequency corresponding to the reduction of the U_{OUT}/U_{IN} ratio to the level of $1/\sqrt{2} \approx 0.707$ the maximum value. The frequency defined in the latter way is easier to calculate for the given RLC components used in the filter circuit.

3.3.1. Low-pass RC filter

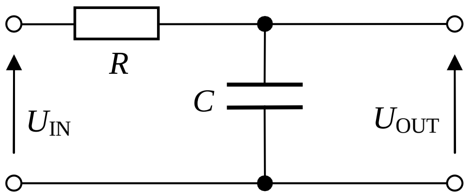

Fig. 2. Circuit diagram of a simple low-pass RC filter.

In the case of the circuit shown in Fig. 2 equation (12) takes the form

$$\frac{\underline{U}_{OUT}}{\underline{U}_{IN}} = \frac{-\dfrac{j}{2\pi fC}}{R - \dfrac{j}{2\pi fC}} . \tag{18}$$

The ratio of the real voltages resulting from eq. (18) describes the amplitude-frequency characteristics of the low-pass RC filter which is plotted in Fig. 3

$$\frac{U_{OUT}}{U_{IN}} = \frac{1}{\sqrt{1+(2\pi fRC)^2}} . \tag{19}$$

Hence, the cut-off frequency of the filter at which $U_{OUT}/U_{IN} = 1/\sqrt{2}$

$$f_c = \frac{1}{2\pi RC} . \tag{20}$$

The phase-frequency characteristic of the low-pass RC filter can be obtained by substituting eq. (18) into (15)

$$\varphi = \mathrm{arctg}(-2\pi fRC) . \tag{21}$$

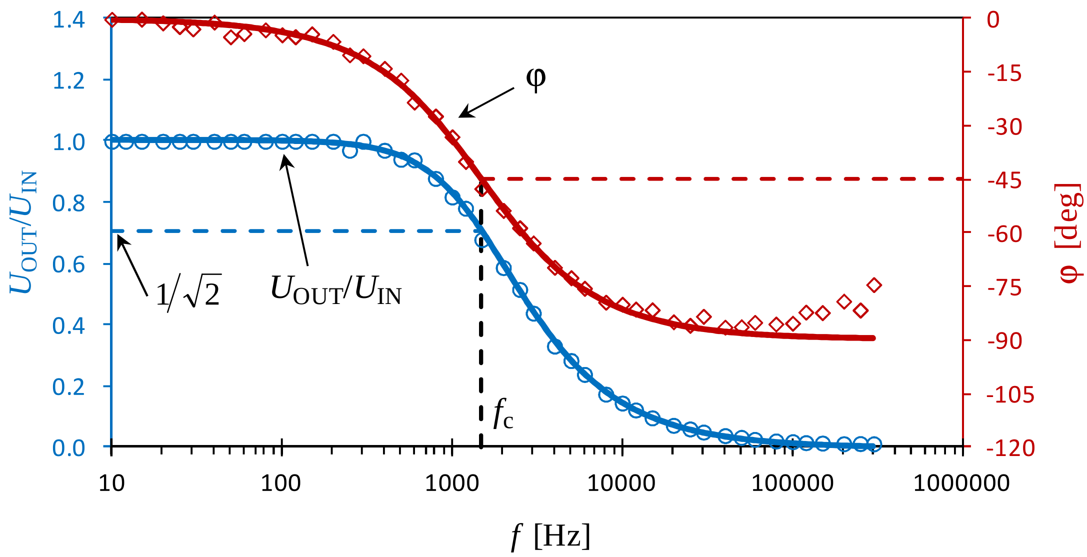

Fig. 3. Amplitude-frequency and phase-frequency characteristics of a simple low-pass RC filter. The points indicate the results of measurements and the solid lines represent theoretical dependencies (19) and (21).

The amplitude-frequency characteristic of the filter plotted according to the formula (19) in the system of two logarithmic axes k [dB] and log(f) has two asymptotes:

horizontal asymptote $k = 0$ dB for $f \to 0$,

oblique asymptote $k = 20\log(f) - 20\log(f_c)$ for $f \to \infty$.

The intersection of the asymptotes corresponds to the cut-off frequency f_c given by the formula (20) for which the attenuation coefficient $k \approx 3$ dB and the phase shift $\varphi = -45°$.

The slope of the oblique asymptote can be expressed in units dB/octave or dB/decade, where the octave means two waveforms with a 2:1 frequency ratio and the decade means a 10:1 ratio. For a simple RC filter, we get respectively 6 dB/octave or 20 dB/decade (Fig. 4).

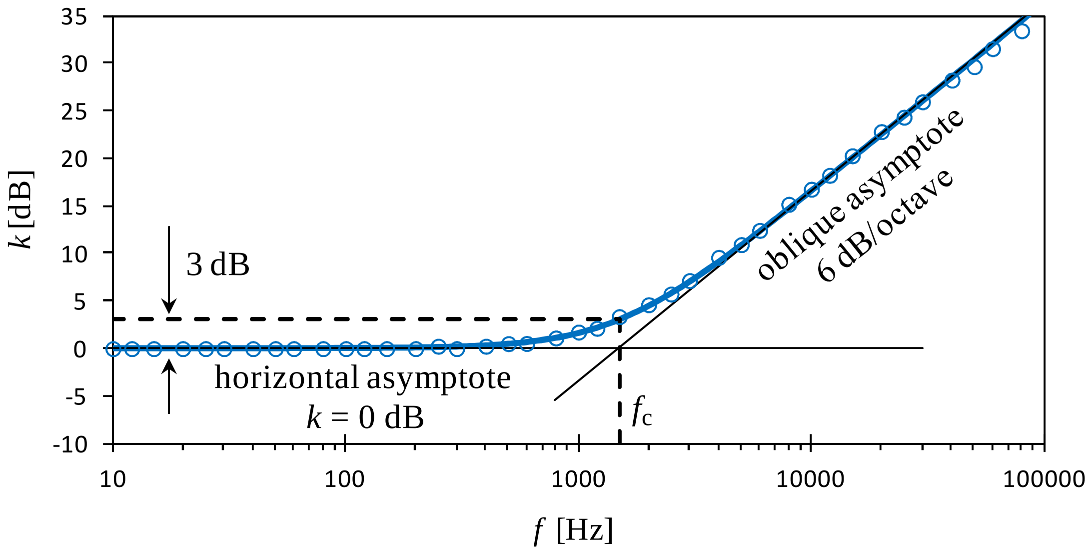

Fig. 4. Amplitude-frequency characteristic of a simple low-pass RC filter plotted using a logarithmic scale on both axes.

3.3.2. High-pass RC filter

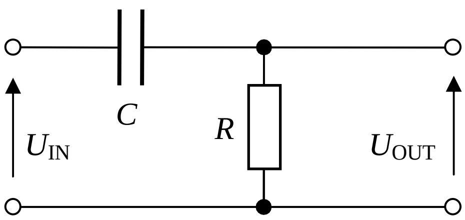

Fig. 5. Circuit diagram of a simple high-pass RC filter.

Equation (12) takes the following form for the circuit shown in Fig. 5

$$\frac{\underline{U}_{OUT}}{\underline{U}_{IN}} = \frac{R}{R - \dfrac{j}{2\pi f C}} \,. \tag{22}$$

The ratio of the real voltages resulting from eq. (22) describes the amplitude-frequency characteristic of the high-pass RC filter shown in Fig. 6

$$\frac{U_{OUT}}{U_{IN}} = \frac{1}{\sqrt{1+\frac{1}{(2\pi fRC)^2}}} \tag{23}$$

The phase-frequency characteristic can be obtained by substituting eq. (22) into (15)

$$\varphi = \text{arctg}\frac{1}{2\pi fRC}. \tag{24}$$

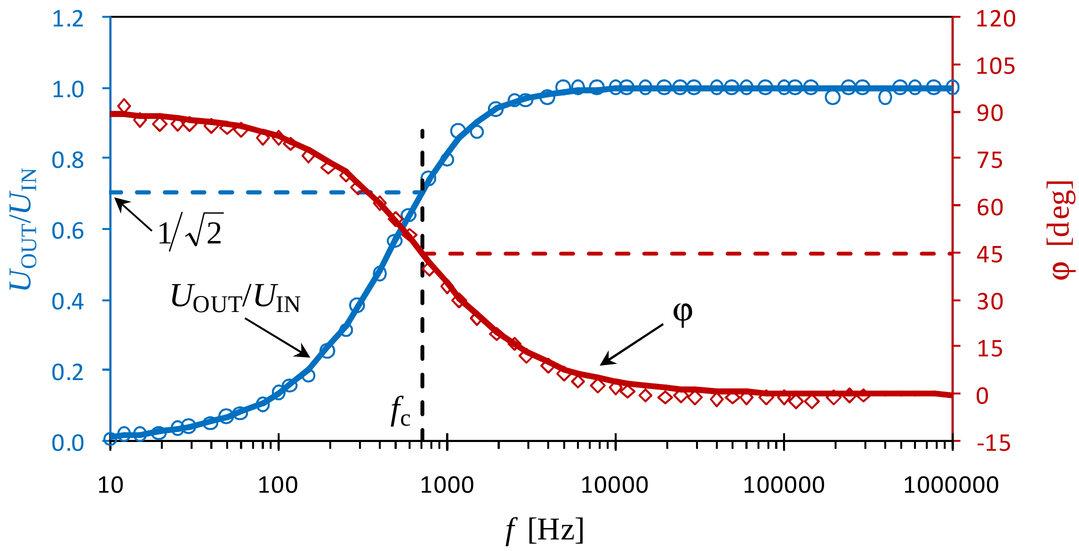

Fig. 6. Amplitude-frequency and phase-frequency characteristics of a simple high-pass RC filter. The points indicate the results of measurements and the solid lines represent theoretical dependencies (23) i (24).

The amplitude-frequency characteristic of the filter plotted according to the formula (23) in the system of two logarithmic axes k [dB] and $\log(f)$ has two asymptotes (Fig. 7), which intersect at the f_c frequency (20):

oblique asymptote $k = -20\log(f) + 20\log(f_c)$ for $f \to 0$,
horizontal asymptote $k = 0$ dB for $f \to \infty$.

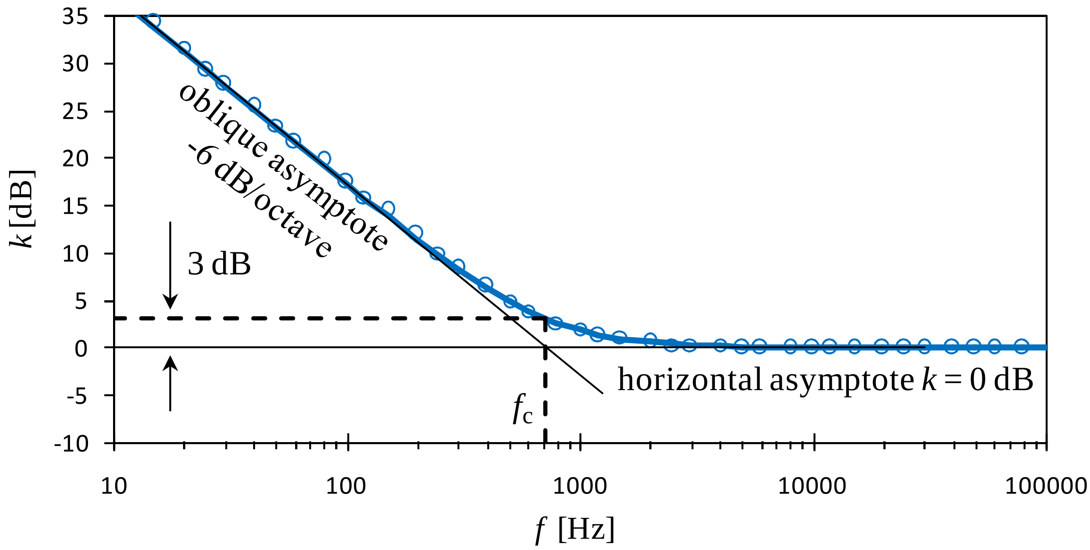

Fig. 7. Amplitude-frequency characteristic of a simple high-pass RC filter plotted using a logarithmic scale on both axes.

3.3.3. Band-pass Wien RC filter

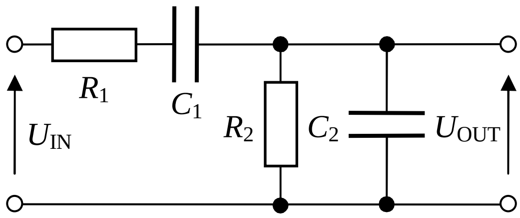

Fig. 8. Circuit diagram of band-pass Wien RC filter.

Equation (12) takes the following form for the circuit shown in Fig. 8

$$\frac{\underline{U}_{OUT}}{\underline{U}_{IN}} = \frac{\left(R_2^{-1} + j2\pi f C_2\right)^{-1}}{R_1 - \dfrac{j}{2\pi f C_1} + \left(R_2^{-1} + j2\pi f C_2\right)^{-1}} . \tag{25}$$

Hence, the ratio of the real output to input voltages (shown in Fig. 9) is given by

$$\frac{U_{OUT}}{U_{IN}} = \left[\left(\frac{R_1}{R_2} + \frac{C_2}{C_1} + 1\right)^2 + \left(2\pi f R_1 C_2 - \frac{1}{2\pi f R_2 C_1}\right)^2\right]^{-1/2} \tag{26}$$

and the phase shift between these voltages

$$\varphi = \mathrm{arctg}\frac{\dfrac{1}{2\pi f R_2 C_1} - 2\pi f R_1 C_2}{\dfrac{R_1}{R_2} + \dfrac{C_2}{C_1} + 1} . \tag{27}$$

The formula (26) indicates that the circuit operates as a band-pass filter and the maximum of its amplitude-frequency characteristic corresponds to the so-called center frequency

$$f_0 = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} \tag{28}$$

and the value of this maximum is

$$\left(\frac{U_{OUT}}{U_{IN}}\right)_{max} = \left(\frac{R_1}{R_2} + \frac{C_2}{C_1} + 1\right)^{-1} . \tag{29}$$

The amplitude-frequency characteristic of the Wien RC filter plotted according to the formula (26) in the system of two logarithmic axes k [dB] and $\log(f)$ has two oblique asymptotes (Fig. 10) that intersect at the point corresponding to the frequency f_0 (28):

oblique asymptote $k = -20\log(f) - 20\log(2\pi R_2 C_1)$ for $f \to 0$,
oblique asymptote $k = +20\log(f) + 20\log(2\pi R_1 C_2)$ for $f \to \infty$.

The phase-frequency characteristic given by the formula (27) starts from $\varphi = +90°$ for $f \to 0$, goes through 0° for the f_0 frequency, and tends to –90° for $f \to \infty$ (Fig. 9).

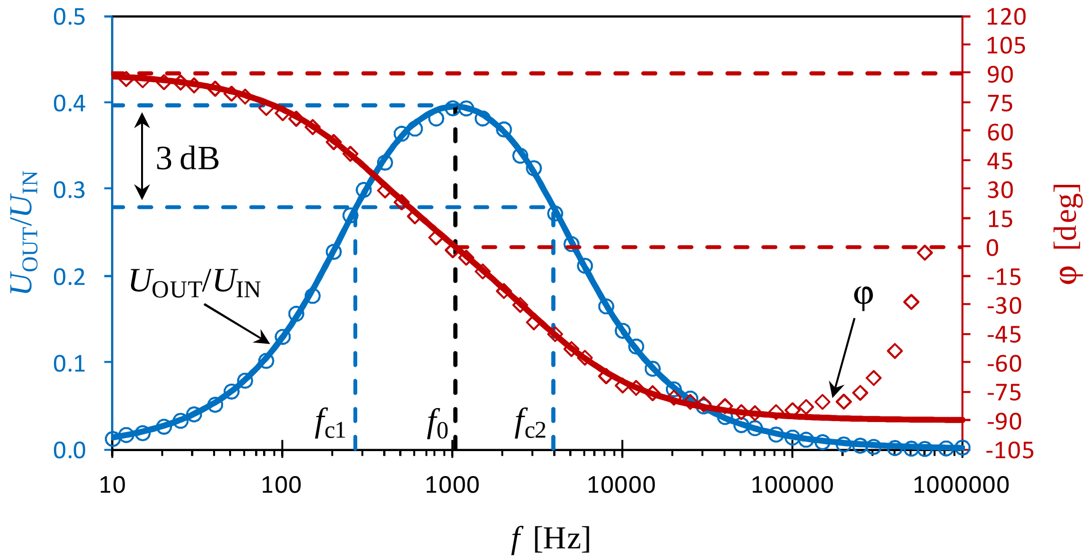

Fig. 9. Amplitude-frequency and phase-frequency characteristics of the Wien RC filter. The points indicate the results of measurements and the solid lines represent theoretical dependencies (26) and (27).

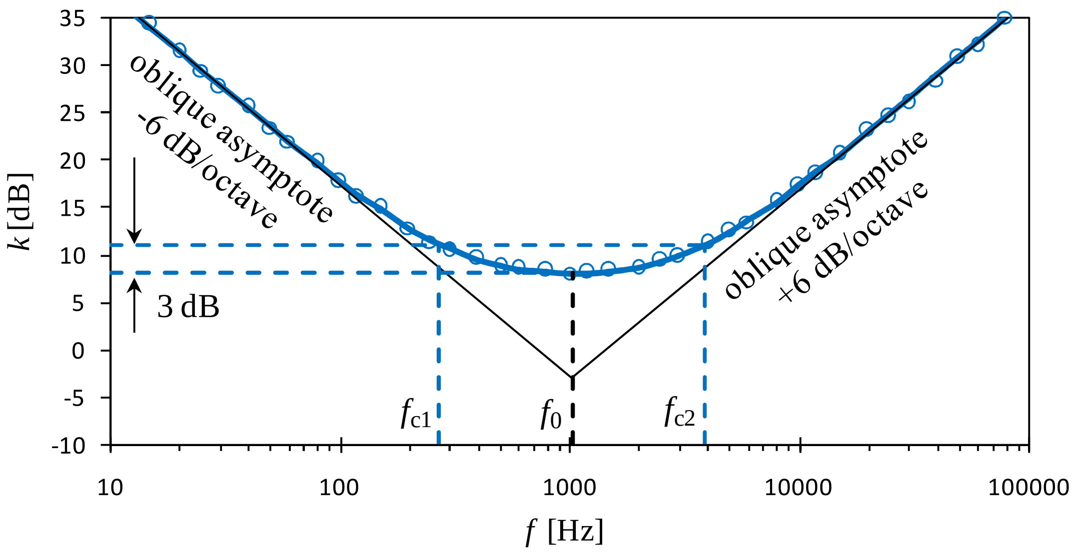

Fig. 10. Amplitude-frequency characteristic of the Wien RC filter plotted using a logarithmic scale on both axes.

The quality factor or Q factor of filter is defined as the ratio of the center frequency f_0 to the bandwidth

$$Q = \frac{f_0}{f_{c2} - f_{c1}}, \tag{30}$$

where f_{c1} and f_{c2} are the lower and upper cut-off frequencies at which the voltage ratio U_{OUT}/U_{IN} decreases to $1/\sqrt{2}$ of the maximum value, which corresponds to an increase in the attenuation k by approximately 3 dB above its minimum. It can be shown using the formulas (26) and (29) that the Q factor (30) of the Wien RC filter is given by

$$Q=\sqrt{\frac{R_1C_2}{R_2C_1}}\left(\frac{R_1}{R_2}+\frac{C_2}{C_1}+1\right)^{-1}=\frac{\sqrt{R_1R_2C_1C_2}}{R_1C_1+R_2C_2+R_2C_1}. \tag{31}$$

The values of Q that can be obtained from the formula (31) are limited to the range 0 ÷ 0.5.

The symbols introduced in formulas (28), (29) and (31) allow to simplify the form of equations (26) and (27) describing the amplitude-frequency and the phase-frequency characteristics

$$\frac{U_{OUT}}{U_{IN}}=\left(\frac{U_{OUT}}{U_{IN}}\right)_{max}\frac{1}{\sqrt{1+Q^2\left(f/f_0-f_0/f\right)^2}}, \tag{32}$$

$$\varphi=\mathrm{arctg}\left[Q\left(f_0/f-f/f_0\right)\right]. \tag{33}$$

3.3.4. Low-pass LC filter

The schematic diagram of a simple low-pass LC filter is shown in Fig. 11.

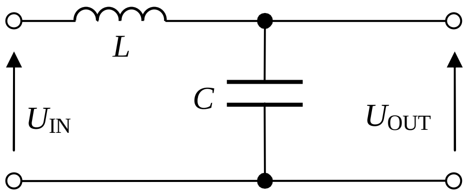

Fig. 11. Circuit diagram of a simple low-pass LC filter.

Resonance in the filter

In a typical real coil, there are significant energy losses due to the series resistance R_L of the coil. This phenomenon is crucial for correct modeling of the filter's characteristics near its resonance frequency. In the low-frequency range (i.e. from 0 to the resonant frequency), the real capacitor can be considered to a good approximation as an ideal capacitor, while at higher frequencies the series resistance of the capacitor R_C may become significant.

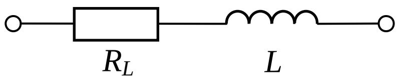

Fig. 12. Equivalent circuit of a real coil.

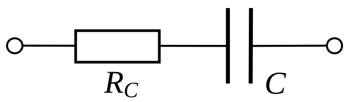

Fig. 13. Equivalent circuit of a real capacitor.

Using the equivalent circuits of the coil and capacitor shown in Figs. 12 and 13, the input impedance of the filter can be written as

$$\underline{Z}_{IN}=\underline{Z}_1+\underline{Z}_2=R_L+R_C+j\left(2\pi fL-\frac{1}{2\pi fC}\right) \tag{34}$$

and the complex current flowing through this impedance

$$\underline{I} = \underline{U}_{\mathrm{IN}} / \underline{Z}_{\mathrm{IN}} \,. \tag{35}$$

When the frequency f is equal to the resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \,, \tag{36}$$

and the load on the filter output is negligible, the reactances X_L and X_C of the L and C elements compensate each other and the input current $I_0 = U_{\mathrm{IN}}/R$ reaches its maximum value limited only by the series resistance $R = R_L + R_C$. Because the same current also flows through the L and C elements $I_0 = U_C / X_C = U_L / X_L$. When the resistance R is small, the voltages on the capacitance U_C and inductance U_L can be even many times higher than the input voltage U_{IN}. The ratio of these voltages at resonance is called *the Q factor of the RLC circuit*

$$Q = \frac{U_L}{U_{\mathrm{IN}}} = \frac{U_C}{U_{\mathrm{IN}}} \tag{37}$$

and

$$Q = \frac{2\pi f_0 L}{R} = \frac{1}{2\pi f_0 RC} = \frac{1}{R}\sqrt{\frac{L}{C}} \,. \tag{38}$$

If U_{IN} = const., it is convenient to express the input current I flowing at any frequency f as a fraction of the resonant current I_0 in the following form

$$\frac{I}{I_0} = \frac{R}{Z_{\mathrm{WE}}} = \frac{1}{\sqrt{1 + Q^2\left(f/f_0 - f_0/f\right)^2}} \,. \tag{39}$$

Considering the family of I/I_0 dependences on f/f_0 given by eq. (39) for various values of the Q parameter, we can see that an increase in the Q value is associated with an increase in the sharpness of the resonance curve. It can be shown that the Q factor defined as the voltage ratio (37) is equivalent to the Q facotor previously defined by formula (30) with the f_0 frequency given by eq. (36) and the frequencies f_{g1} and f_{g2} corresponding to $I/I_0 = 1/\sqrt{2}$.

In practice, a function generator with an output resistance of 50 Ω is not able to maintain a constant voltage at the filter input, where R is also of the order of several dozen ohms. If the amplitude control knob in the generator remains in a fixed position, the greatest voltage drop can be observed at the frequency $f = f_0$.

<u>Signal transmission through the low-pass LC filter</u>

Equation (12) takes the following form for the circuit shown in Fig. 11

$$\frac{\underline{U}_{\mathrm{OUT}}}{\underline{U}_{\mathrm{IN}}} = \frac{-\dfrac{j}{2\pi fC} + R_C}{j\left(2\pi fL - \dfrac{1}{2\pi fC}\right) + R_L + R_C} = \frac{(Qf_0/f)(-j + 2\pi f R_C C)}{jQ(f/f_0 - f_0/f) + 1} \,. \tag{40}$$

Hence, the ratio of the real output to input voltages (shown in Fig. 14) is given by

$$\frac{U_{\mathrm{OUT}}}{U_{\mathrm{IN}}} = \frac{Qf_0\sqrt{1 + (2\pi f R_C C)^2}}{f\sqrt{1 + Q^2\left(f/f_0 - f_0/f\right)^2}} \tag{41}$$

and the phase shift between the output and input voltages

$$\varphi = -\arccos \frac{-Q(f/f_0 - f_0/f) + 2\pi f R_C C}{\sqrt{1+Q^2(f/f_0 - f_0/f)^2}\sqrt{1+(2\pi f R_C C)^2}}. \tag{42}$$

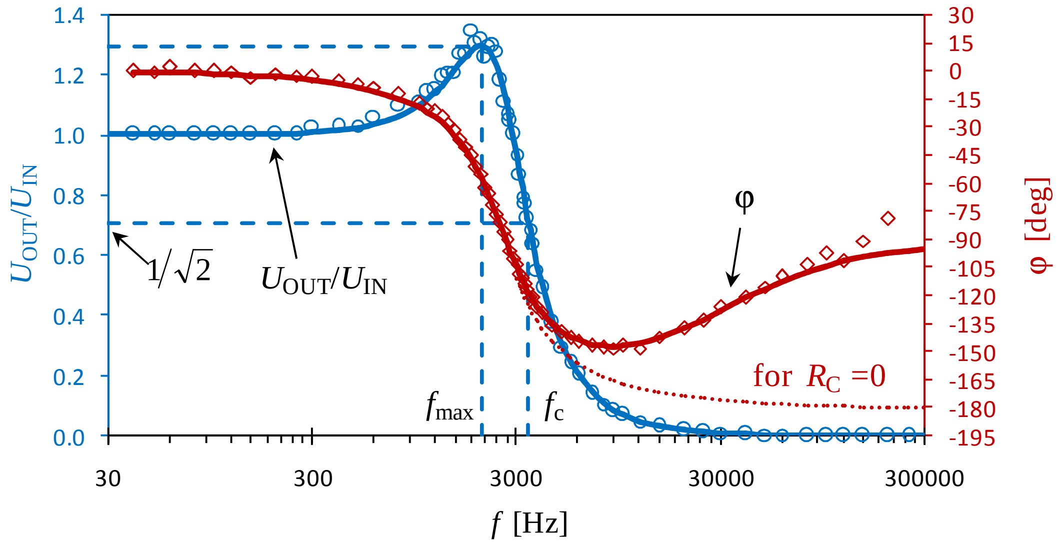

Fig. 14. Amplitude frequency and phase frequency characteristics of a simple low-pass LC filter. The points indicate the results of measurements and the solid lines represent theoretical dependencies (41) and (42).

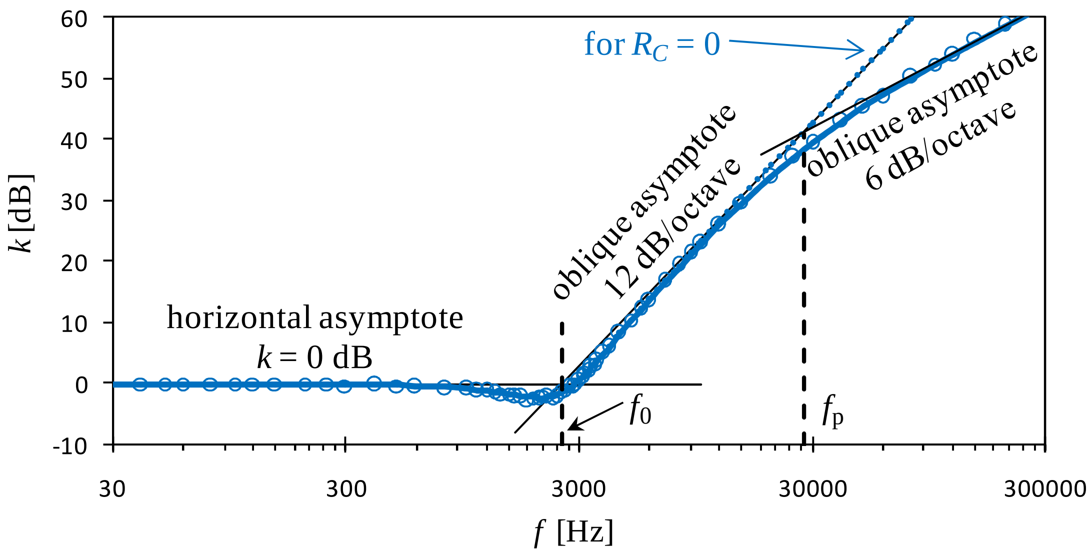

Fig. 15. Amplitude-frequency characteristic of a simple low-pass LC filter plotted using a logarithmic scale on both axes.

The amplitude-frequency characteristic plotted according to the formula (41) for a filter with an ideal capacitor ($R_C = 0$) has two asymptotes in the system of two logarithmic axes k [dB] and log(f): a horizontal asymptote and an oblique asymptote with a slope of 12 dB/octave (Fig. 15). In the case of a real filter ($R_C > 0$), the part of the characteristic with an almost constant slope of 12 dB/octave = 40 dB/decade is limited and at high frequencies for which $X_C << R_C$ the characteristic tends to an asymptote with a slope of 6 dB/octave = 20 dB/decade:

horizontal asymptote	$k = 0$ dB	for $f \to 0$,	
oblique asymptote	$k = 40 \log(f/f_0)$	for $R_C << X_C << X_L$,	(43)
oblique asymptote	$k = 20 \log(f) - 20\log(R_C/2\pi L)$	for $f \to \infty$.	

The horizontal asymptote and the 12 dB/octave asymptote intersect at the resonant frequency f_0 (36), while the asymptotes 12 dB/octave and 6 dB/octave intersect at the f_p frequency

$$f_\mathrm{p} = \frac{1}{2\pi R_C C} \tag{44}$$

which corresponds to the equality of the capacitor's reactance $1/(2\pi f_\mathrm{p} C)$ and its resistance R_C.

In the case of a filter with an ideal capacitor ($R_C = 0$), the phase-frequency characteristic given by the formula (42) starts from $\varphi = 0°$ for $f \to 0$, passes through $-90°$ for the f_0 frequency, and tends to $-180°$ for $f \to \infty$ (Fig. 14). However, the resistance $R_C > 0$ causes that the phase shift reach a minimum in the range $-180° \div -90°$ and increases with further increase in frequency.

The voltage ratio (41) reaches values greater than one at frequencies close to f_0. It is worth noting, however, that the output voltage U_OUT is proportional to the current I flowing in the resonant circuit and to the capacitor impedance, which decreases with increasing frequency. Hence, the maximum of the $U_\mathrm{OUT}/U_\mathrm{IN}$ voltage ratio occurs at a frequency f_max lower than the frequency f_0 corresponding to the maximum current I. The complicated $U_\mathrm{OUT}/U_\mathrm{IN}$ on f dependence makes it difficult to derive an exact formula for the f_max frequency. However, near the f_max frequency we can neglect the term containing R_C in eq. (41), which simplifies the analytical derivation of f_max

$$f_\mathrm{max} \approx f_0 \sqrt{1 - \frac{1}{2Q^2}} \,. \tag{45}$$

The maximum voltage ratio at $f = f_\mathrm{max}$ is

$$\left(\frac{U_\mathrm{OUT}}{U_\mathrm{IN}}\right)_\mathrm{max} \approx \frac{Q}{\sqrt{1 - \frac{1}{4Q^2}}} \,. \tag{46}$$

Hence, the Q factor corresponding to the measured value $(U_\mathrm{OUT}/U_\mathrm{IN})_\mathrm{max}$ can be calculated as

$$Q \approx \left(\frac{U_\mathrm{OUT}}{U_\mathrm{IN}}\right)_\mathrm{max} \sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{1 - \left(\frac{U_\mathrm{OUT}}{U_\mathrm{IN}}\right)_\mathrm{max}^{-2}}} \,. \tag{47}$$

3.3.5. High-pass LC filter (*extended version*)

The schematic diagram of a simple high-pass LC filter is shown in Fig. 16.

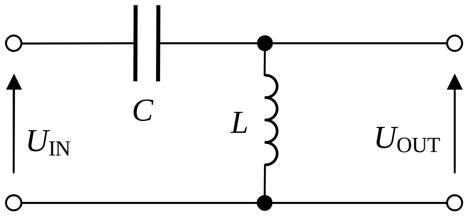

Fig. 16. Circuit diagram of a simple high-pass LC filter.

Considering again the equivalent circuits of the coil and capacitor shown in Figs. 12 and 13, the following formulas remain valid: the filter input impedance (34), the frequency f_0 (36) corresponding to the maximum current, the Q factor (37) and (38), and the current ratio I/I_0 (39).

Equation (12) for the circuit under consideration has the following form

$$\frac{\underline{U}_{OUT}}{\underline{U}_{IN}} = \frac{j\,2\pi fL + R_L}{j\left(2\pi fL - \dfrac{1}{2\pi fC}\right) + R_L + R_C} = \frac{(Qf/f_0)(j + R_L/2\pi fL)}{jQ(f/f_0 - f_0/f) + 1}. \tag{48}$$

Hence, the ratio of the real output to input voltages (shown in Fig. 17) is given by

$$\frac{U_{OUT}}{U_{IN}} = \frac{Qf\sqrt{1+(R_L/2\pi fL)^2}}{f_0\sqrt{1+Q^2(f/f_0 - f_0/f)^2}} \tag{49}$$

and the phase shift between the output and input voltages

$$\varphi = \arccos\frac{Q(f/f_0 - f_0/f) + R_L/2\pi fL}{\sqrt{1+Q^2(f/f_0 - f_0/f)^2}\sqrt{1+(R_L/2\pi fL)^2}}. \tag{50}$$

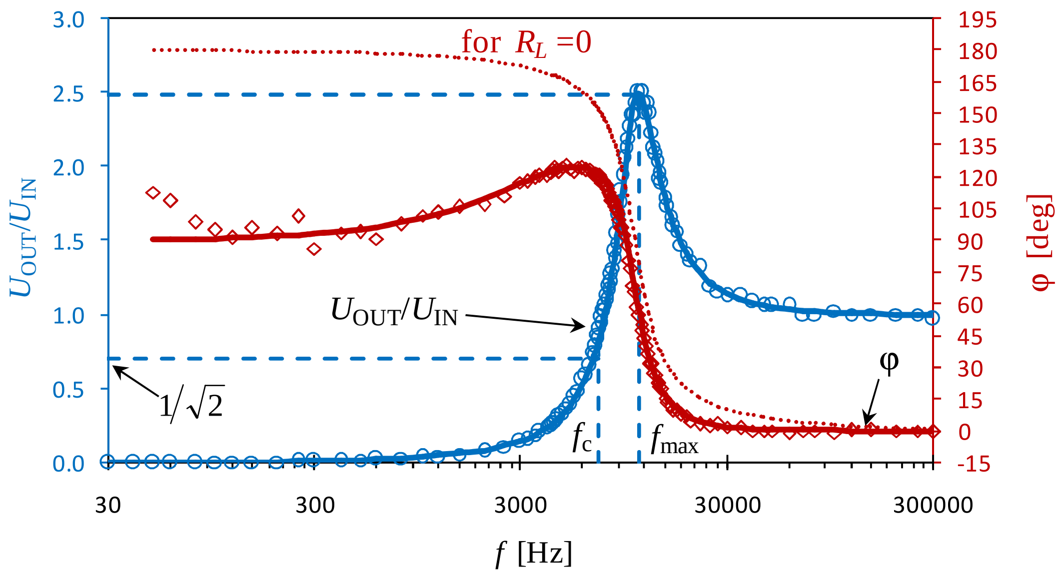

Fig. 17. Amplitude-frequency and phase-frequency characteristics of a simple high-pass LC filter. The points indicate the results of measurements and the solid lines represent theoretical dependencies (49) and (50).

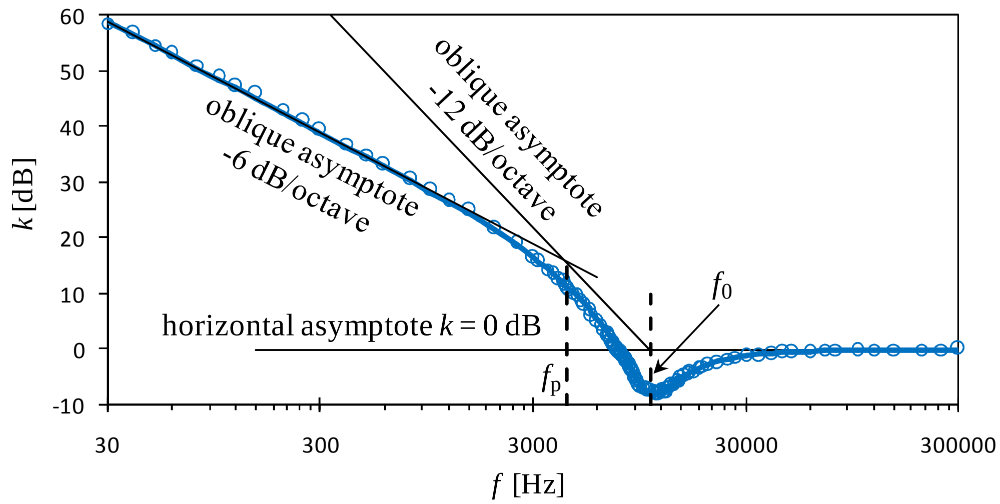

Fig. 18. Amplitude-frequency characteristic of a simple high-pass LC filter plotted using a logarithmic scale on both axes.

The amplitude-frequency characteristic plotted according to the formula (49) for a filter with an ideal coil ($R_L = 0$) has two asymptotes in the system of two logarithmic axes k [dB] and log(f): a horizontal asymptote and an oblique asymptote –12 dB/octave. In a real circuit, where $R_L > 0$, the coil resistance R_L dominates over the reactance $X_L = 2\pi fL$ at low frequencies and the LC high-pass filter works as an RC high-pass filter with a characteristic asymptote –6 dB/octave = –20 dB/dekadę (Fig. 18):

oblique asymptote	$k = -20\log(f) - 20\log(2\pi R_L C)$	for $f \to 0$,
oblique asymptote	$k = -40\log(f/f_0)$	for $R_L << X_L << X_C$,
horizontal asymptote	$k = 0$ dB	for $f \to \infty$.

The horizontal asymptote and the –12 dB/octave asymptote intersect at the frequency f_0 (36), while the asymptotes –6 dB/octave and –12 dB/octave intersect at the f_p frequency

$$f_p = \frac{R_L}{2\pi L} \tag{51}$$

which corresponds to the equality of the coil's reactance $2\pi f_p L$ and its resistance R_L.

In the case of a filter with an ideal coil ($R_L = 0$), the phase-frequency characteristic given by formula (50) decreases monotonically from 180° for $f \to 0$ to 0° for $f \to \infty$ (Fig. 17). However, the resistance $R_L > 0$ causes that the characteristic starts from 90° for $f \to 0$, increase to a certain maximum value less than 180° and drop to 0° for $f \to \infty$.

WARNING: in the circuit tested in this exercise, the $R_L << X_L << X_C$ relationship is not well satisfied in any frequency range and consequently the theoretical asymptote –12 dB/octave visible in Fig. 18 is clearly shifted relative to the measurement points. This result well illustrates the difficulties encountered when designing LC filters with a slope of –12 dB/octave over a wide frequency range. In practice, this task is much simpler when using active RC filters.

The output voltage U_{OUT} is proportional to the current I flowing in the resonant circuit and to the coil impedance, which increases with increasing frequency. Hence, the maximum voltage ratio U_{OUT}/U_{IN} occurs at a frequency f_{max} higher than the frequency f_0 corresponding to the maximum current I. Because the effect of the R_L resistance cannot be neglected, determining the theoretical value of f_{max} is complicated and beyond the scope of this exercise.

3.3.6. Band-pass Wien LC filter (*extended version*)

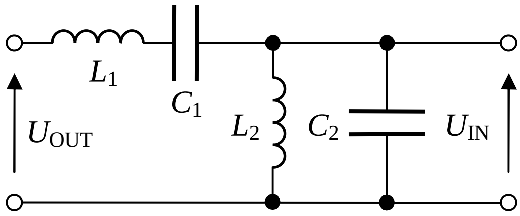

Fig. 19. Circuit diagram of Wien LC filter.

Equation (12) takes the following form for the Wien LC filter shown in Fig. 19 composed of coils and capacitors with the equivalent circuits shown in Figs. 12 and 13

$$\frac{\underline{U}_{\mathrm{OUT}}}{\underline{U}_{\mathrm{IN}}}=\frac{\left\{\left(j2\pi fL_2+R_{L2}\right)^{-1}+\left[-j/\left(2\pi fC_2\right)+R_{C2}\right]^{-1}\right\}^{-1}}{j\left(2\pi fL_1-\dfrac{1}{2\pi fC_1}\right)+R_{L1}+R_{C1}+\left\{\left(j2\pi fL_2+R_{L2}\right)^{-1}+\left[-j/\left(2\pi fC_2\right)+R_{C2}\right]^{-1}\right\}^{-1}}\,. \quad (52)$$

Because equation (52) is difficult to analyze, a complete quantitative analysis will only be presented for the case of a filter composed of ideal coils and capacitors. Substitution of $R_{L1} = R_{L2} = R_{C1} = R_{C2} = 0$ simplifies formula (52) to the form

$$\frac{\underline{U}_{\mathrm{OUT}}}{\underline{U}_{\mathrm{IN}}}=\frac{1}{\sqrt{\dfrac{L_1C_2}{L_2C_1}}\left(\dfrac{f}{f_1}-\dfrac{f_1}{f}\right)\left(\dfrac{f_2}{f}-\dfrac{f}{f_2}\right)+1}\,, \quad (53)$$

where f_1 and f_2 are the resonance frequencies

$$f_1=\frac{1}{2\pi\sqrt{L_1C_1}}\,,\qquad f_2=\frac{1}{2\pi\sqrt{L_2C_2}}\,. \quad (54)$$

The ratio $\underline{U}_{\mathrm{OUT}}/\underline{U}_{\mathrm{IN}}$ given by formula (53) does not contain an imaginary part and takes positive values around the f_0 frequency and negative values for other frequencies. An exemplary dependence of the module of expression (53) on the frequency f is presented in Fig. 20. Regardless of the choice of the L_1, L_2, C_1 and C_2 values, this module always has one local minimum at the frequency

$$f_0=\sqrt{f_1 f_2}\,, \quad (55)$$

which is surrounded by two maxima $U_{\mathrm{OUT}}/U_{\mathrm{IN}} \to \infty$. These maxima occur at frequencies that do not coincide with f_1 nor f_2.

The phase shift resulting from the $\underline{U}_{\mathrm{OUT}}/\underline{U}_{\mathrm{IN}}$ expression given by formula (53) takes only the values $\varphi = 0°$ around the frequency f_0 or $\varphi = \pm 180°$ (Fig. 20). The values +180° and −180° are indistinguishable in measurements, but distinguishable on the basis of a theoretical analysis of the limit as $R_{L1} = R_{L2} = R_{C1} = R_{C2} \to 0$.

The expression (52) becomes complex for the resistances R_{L1}, R_{L2}, R_{C1}, R_{C2} greater than zero. Increasing the resistances initially leads to a decrease in the height of the $U_{\mathrm{OUT}}/U_{\mathrm{IN}}$ maximums and to smoother switching of the φ value (Fig. 21). Further increasing these resistances or selecting other values of L and C may lead to the disappearance of the maximums and to the transition of the minimum at frequency f_0 into a single maximum (Fig. 22). In the case of circuits available in the Laboratory of electronics, both of these cases may occur for different settings of the S2 switch in experimental module.

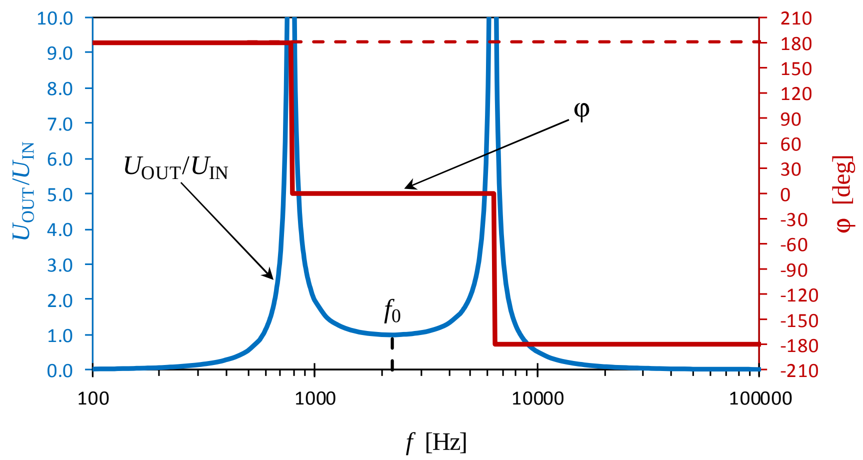

Fig. 20. An example of the theoretical amplitude-frequency and phase-frequency characteristics of a Wien LC filter composed of ideal coils and capacitors.

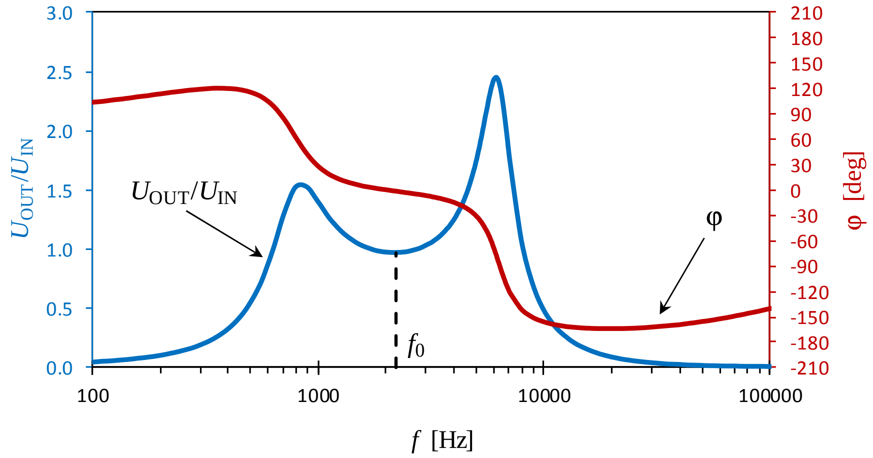

Fig. 21. An example of the theoretical amplitude-frequency and phase-frequency characteristics of the Wien LC filter for small values of series resistances of coils and capacitors compared to their reactances at frequency f_0.

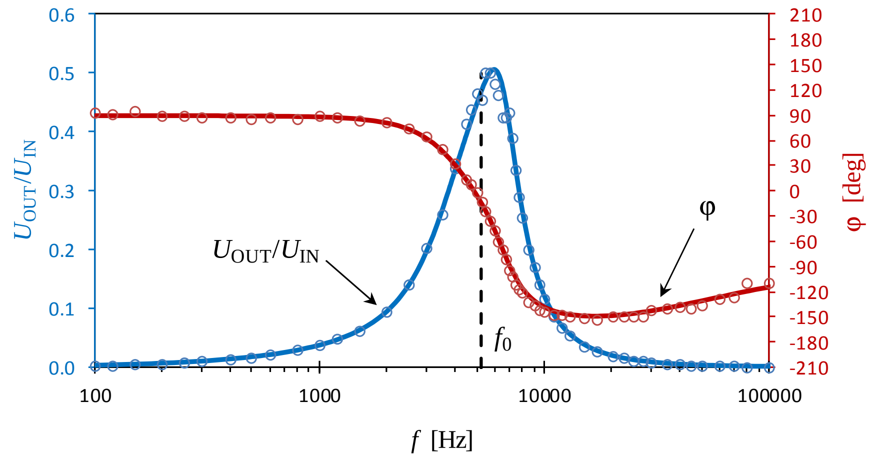

Fig. 22. An example of amplitude-frequency and phase-frequency characteristics of the Wien LC filter for high values of series resistances of the coils. The points indicate the results of measurements and the solid lines represent theoretical dependencies.

4. Available equipment

4.1. Experimental module

The experimental module consists of two parts (Fig. 23):
- the upper part containing R_1, R_2, C_1, C_2 elements with sockets that allow connection of RC filters, and the switch S1 to select various values of C_1 and C_2 (given in Table 2),
- the lower part containing L_1, L_2, C_1, C_2 elements with sockets that allow connection of LC filters, and the switch S2 to select various values of L_2, C_1 and C_2 (given in Table 3).

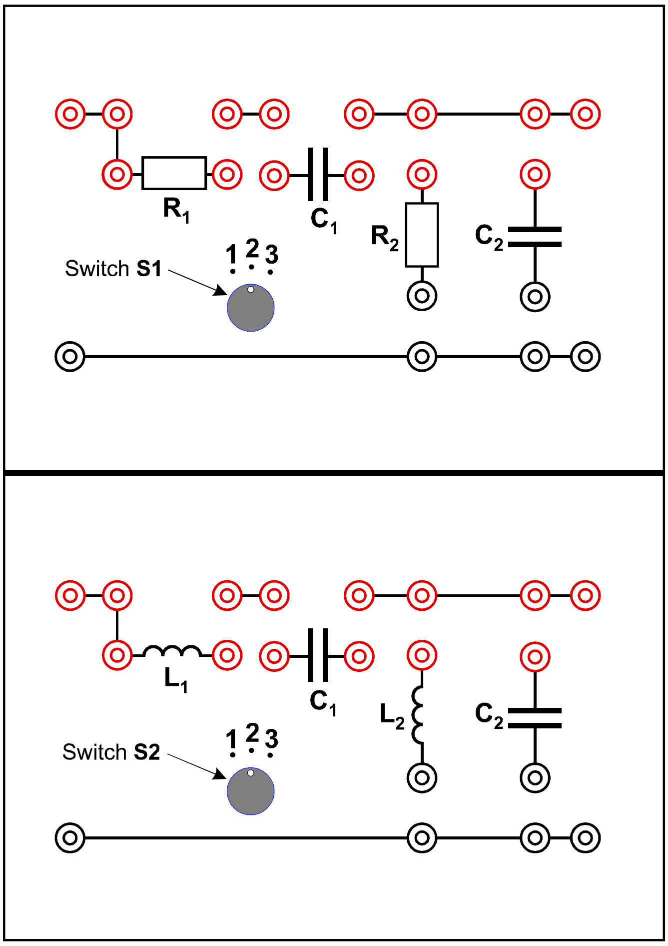

Fig. 23. The front panel of the experimental module.

4.2. Function generator

Function generator model DF1641B [10].

4.3. Oscilloscope

The dual-channel digital oscilloscope SIGLENT SDS1052DL [10] is used to observe waveforms at the input and output of filters. The oscilloscope also displays numerical values of voltages, phase shift and frequency.

5. Experimental procedure

5.1. Amplitude-frequency and phase-frequency characteristics of passive low-pass RC filter

1. Connect the OUTPUT of the function generator **G** with the RC voltage divider input as shown in Fig. 24 and with CH1 input of the oscilloscope using the T-type connector. Use BNC–BNC cable for connecting the generator with oscilloscope and BNC–banana-plug cable for connecting the generator with the input of experimental circuit.
2. Connect the output of experimental circuit with CH2 input of the oscilloscope as show in Fig. 24 using the BNC–banana-plug cable.
3. After supervisor's approval, turn on the power of the devices. Select position of the switch S1 in the experimental module according to recommendations of supervisor.
4. Select the sine waveform (without the DC OFFSET) in the generator and set the frequency to 30 Hz and the voltage to 20.0 $V_{p\text{-}p}$ on the displays in the generator.
5. Before starting the work with the digital oscilloscope it is recommended to press the DEFAULT SETUP button. Then switch the oscilloscope to work in two-channel mode (both CH1 and CH2 buttons must be illuminated) with the AC coupling mode in each channel. After pressing the TRIG MENU button, select the triggering with the signal applied to the CH1 input. Adjust the oscilloscope settings to obtain an optimal view of both waveforms.
6. Press the MEASURE button to display a menu of measurement parameters. Then, using the buttons on the right side of the screen, change the current settings to the following: the root mean square voltage Vrms in channel CH1 (U_{IN} in Table 1), Vrms in channel CH2 (U_{OUT}) and the phase difference CH1-CH2 ($\varphi_{CH1\text{-}CH2}$ in Table 1).
7. Measure the input voltage U_{IN}, the output voltage U_{OUT} and the phase difference $\varphi_{CH1\text{-}CH2}$ as functions of frequency f in the range 30 Hz ÷ 300 kHz. The optimal step of frequency changing should increase more or less proportionally to the frequency already achieved. In case of sudden changes in the measured values, reduce the frequency changes.
   WARNING: When changing the frequency, remember to adjust the oscilloscope settings to get the optimal waveforms display before reading the measurement results.
8. Record the obtained measurement results f, U_{IN}, U_{OUT}, $\varphi_{CH1\text{-}CH2}$, the V/DIV gain settings, and the position of the switch S1 in Table 1.

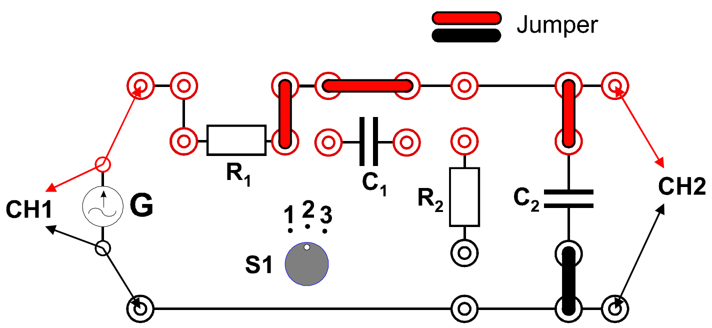

Fig. 24. Scheme of connection diagram for determining the amplitude-frequency and phase-frequency characteristics of passive low-pass RC filter.

Table 1. The measurement results obtained for .... filter with the S1/S2 switch in position ....

f		Δf	U_{IN}		ΔU_{IN}	U_{OUT}		ΔU_{OUT}	$\varphi_{CH1\text{-}CH2}$	$\varphi_{OUT\text{-}IN}$	U_{OUT}/U_{IN}	$\Delta(U_{OUT}/U_{IN})$	k	Δk
[Hz]	[kHz]	[kHz]	[V/DIV]	[V]	[V]	[V/DIV]	[V]	[V]	[deg]	[deg]			[dB]	[dB]

WARNING: only necessary data should be recorded during measurements: f [Hz or kHz], U_{IN} [V/DIV], U_{IN} [V], U_{OUT} [V/DIV], U_{IN} [V], $\varphi_{CH1\text{-}CH2}$ [deg].
The remaining results will be calculated during the preparation of the report (see Chapter 6, item 6).

5.2. Amplitude-frequency and phase-frequency characteristics of passive high-pass RC filter

1. Connect the OUTPUT of the function generator **G** with the RC voltage divider input as shown in Fig. 25 and with CH1 input of the oscilloscope using the T-type connector. Use BNC–BNC cable for connecting the generator with oscilloscope and BNC–banana-plug cable for connecting the generator with the input of experimental circuit.
2. Connect the output of experimental circuit with CH2 input of the oscilloscope as show in Fig. 25 using the BNC–banana-plug cable.
3. Prepare a new copy of Table 1 with an appropriate description of the filter.
4. Repeat the measurement procedure described in points 3 ÷ 8 of section 5.1 above.

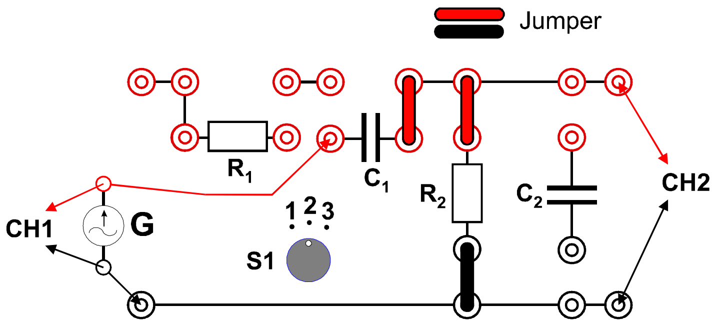

Fig. 25. Scheme of connection diagram for determining the amplitude-frequency and phase-frequency characteristics of passive high-pass RC filter.

5.3. Amplitude-frequency and phase-frequency characteristics of passive band-pass Wien RC filter

1. Connect the OUTPUT of the function generator **G** with the RC voltage divider input as shown in Fig. 26 and with CH1 input of the oscilloscope using the T-type connector. Use BNC–BNC cable for connecting the generator with oscilloscope and BNC–banana-plug cable for connecting the generator with the input of experimental circuit.
2. Connect the output of experimental circuit with CH2 input of the oscilloscope as show in Fig. 26 using the BNC–banana-plug cable.
3. Prepare a new copy of Table 1 with an appropriate description of the filter.
4. Repeat the measurement procedure described in points 3 ÷ 8 of section 5.1.
   WARNING: special attention should be paid to increasing the density of measurements near the frequency for which the ratio U_{OUT}/U_{IN} reaches the maximum and $\varphi_{CH1\text{-}CH2} = 0°$.

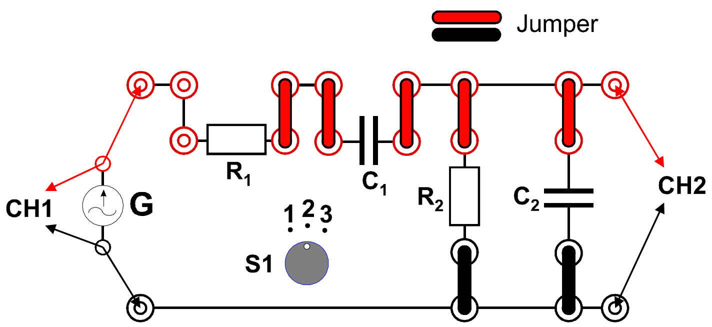

Fig. 26. Scheme of connection diagram for determining the amplitude-frequency and phase-frequency characteristics of passive band-pass Wien RC filter.

5.4. Amplitude-frequency and phase-frequency characteristics of passive low-pass LC filter

1. Connect the OUTPUT of the function generator **G** with the LC voltage divider input as shown in Fig. 27 and with CH1 input of the oscilloscope using the T-type connector. Use BNC–BNC cable for connecting the generator with oscilloscope and BNC–banana-plug cable for connecting the generator with the input of experimental circuit.
2. Connect the output of experimental circuit with CH2 input of the oscilloscope as show in Fig. 27 using the BNC–banana-plug cable.
3. Prepare a new copy of Table 1 with an appropriate description of the filter.
4. Repeat the measurement procedure described in points 3 ÷ 8 of section 5.1.
   WARNING: special attention should be paid to increasing the density of measurements near the frequency for which the ratio U_{OUT}/U_{IN} reaches the maximum.

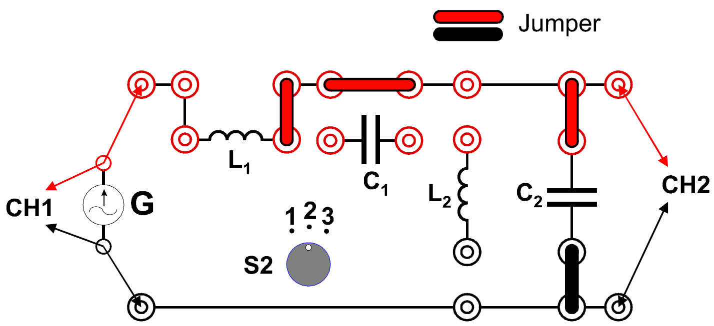

Fig. 27. Scheme of connection diagram for determining the amplitude-frequency and phase-frequency characteristics of passive low-pass LC filter.

5.5. Amplitude-frequency and phase-frequency characteristics of passive high-pass LC filter (*extended version*)

1. Connect the OUTPUT of the function generator **G** with the LC voltage divider input as shown in Fig. 28 and with CH1 input of the oscilloscope using the T-type connector. Use BNC–BNC cable for connecting the generator with oscilloscope and BNC–banana-plug cable for connecting the generator with the input of experimental circuit.
2. Connect the output of experimental circuit with CH2 input of the oscilloscope as show in Fig. 28 using the BNC–banana-plug cable.
3. Prepare a new copy of Table 1 with an appropriate description of the filter.
4. Repeat the measurement procedure described in points 3 ÷ 8 of section 5.1.
   WARNING: special attention should be paid to increasing the density of measurements near the frequency for which the ratio U_{OUT}/U_{IN} reaches the maximum.

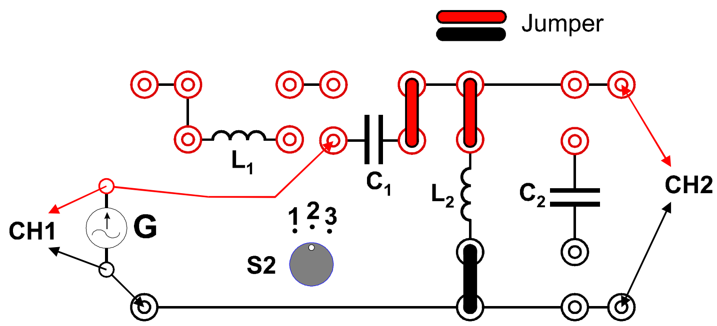

Fig. 28. Scheme of connection diagram for determining the amplitude-frequency and phase-frequency characteristics of passive high-pass LC filter..

5.6. Amplitude-frequency and phase-frequency characteristics of passive band-pass Wien LC filter (*extended version*)

1. Connect the OUTPUT of the function generator **G** with the LC voltage divider input as shown in Fig. 29 and with CH1 input of the oscilloscope using the T-type connector. Use BNC–BNC cable for connecting the generator with oscilloscope and BNC–banana-plug cable for connecting the generator with the input of experimental circuit.
2. Connect the output of experimental circuit with CH2 input of the oscilloscope as show in Fig. 29 using the BNC–banana-plug cable.
3. Prepare a new copy of Table 1 with an appropriate description of the filter.
4. Repeat the measurement procedure described in points 3 ÷ 8 of section 5.1, but now the frequency can be limited to the range 100 Hz ÷ 100 kHz.
   WARNING: before measurements, it is recommended to initially search for the maxima of the amplitude-frequency characteristic (one or two maxima are possible). Special attention should be paid to increasing the density of measurements near the frequencies for which the ratio U_{OUT}/U_{IN} reaches maxima.

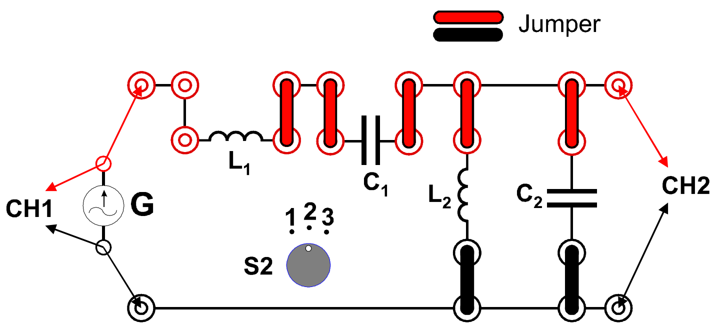

Fig. 29. Scheme of connection diagram for determining the amplitude-frequency and phase-frequency characteristics of passive band-pass Wien LC filter.

6. Report elaboration

Report has to be composed of:

1. Front page (by using a template).
2. Description of experiment purposes.
3. Short introduction.
   The introduction should contain basic definitions and the formulas used in calculations. Each formula given in the introduction as well as in the following parts of the report must be accompanied by a sequence number.
4. Schematic diagrams of tested circuits.
   The report should contain only diagrams of the systems, which were actually compiled during the measurements. Each scheme must be accompanied by a sequence number and title. All the components shown in the diagram must be clearly described and identified using commonly used symbols.
5. List of used instruments and devices (id/stock number, type, setting and range values).
6. Results of measurement in the form of tables and equipment settings.
   Each table must be accompanied by a sequence number and title. In addition to a copy of the data recorded during the measurements, the tables should also include:
   6.1. The maximum uncertainty Δf of each measured frequency f calculated according to the user manual for the function generator [10] (see the counter's accuracy) or appendix A8 in the Polish version of the instruction for the exercise E01 "Miernictwo", subject "Podstawy elektroniki (laboratorium)", available at fizyka.p.lodz.pl/dla-studentow/informatyka/podstawy-elektroniki-laboratorium/.
   6.2. The maximum uncertainties ΔU_{IN} and ΔU_{OUT} of measured voltages U_{IN} and U_{OUT} calculated according to the user manual for the oscilloscope [10] or appendix A6 in the Polish version of the instruction for the exercise E01 "Miernictwo".
   6.3. The values of $\varphi_{CH1\text{-}CH2}$ recorded from the oscilloscope screen, which mean the phase of the signal at the filter input (CH1) relative to the output (CH2) and are in the range of $0 \div 360°$, must be negated to obtain the output to input phase difference and then reduced to the range of $-180° \div +180°$. The results of these calculations should be written in the table as $\varphi_{OUT\text{-}IN}$. The reduction consists in rewriting values from $-180°$ to $+180°$ without changing, the values smaller than $-180°$ should be reduced to this range by adding 360°, and the remaining values greater than $+180°$ should be reduced by subtracting 360°. When using the MS EXCEL spreadsheet, the required calculation can be made using the formula:

      ```
      =MOD(180 – C4;360) – 180
      ```

      where C4 is an example cell address with the value of $\varphi_{CH1\text{-}CH2}$.
      WARNING: only the $\varphi_{OUT\text{-}IN}$ values are directly comparable with the theoretical predictions for the phase shift given in chapters 3.3.1 ÷ 3.3.6.
   6.4. The results of calculations of the ratio U_{OUT}/U_{IN} and the attenuation coefficient k according to the formula (17).
   6.5. The maximum uncertainties $\Delta(U_{OUT}/U_{IN})$ and Δk of indirectly determined values. When making calculations, it should be noted that the propagation of maximum uncertainties, in contrast to standard uncertainties, is described by complete differential equation (see e.g. in [11], chapter II.2.5. or in [12], chapter 7.6). If the measured quantity is a known function $y = y(x_1, x_2, \ldots, x_N)$ of several x_i variables measured directly that are subjected to a rectangular distributions of probabilities

with maximum measurement uncertainties Δx_i, the total maximum uncertainty Δy can be calculated according to the formula:

$$\Delta y = \sum_{i=1}^{N} \left| \frac{\partial y}{\partial x_i} \right| \Delta x_i \, . \tag{56}$$

Hence, for the U_{OUT}/U_{IN} voltage ratio, we get

$$\Delta\left(U_{OUT}/U_{IN}\right) = \frac{U_{OUT}}{U_{IN}} \left(\frac{\Delta U_{OUT}}{U_{OUT}} + \frac{\Delta U_{IN}}{U_{IN}} \right), \tag{57}$$

and in the case of the attenuation coefficient k defined by the formula (17)

$$\Delta k = \frac{20}{\ln 10} \, \frac{\Delta\left(U_{OUT}/U_{IN}\right)}{U_{OUT}/U_{IN}} \, . \tag{58}$$

The obtained numerical results should be written with the appropriate precision. This problem has been described in detail, for example in chapter II.2.2 of the script [11].

7. Plots and analysis of obtained results.
   7.1. Plot the following characteristics for each filter:
   – amplitude-frequency characteristic $U_{OUT}/U_{IN}(f)$,
   – phase-frequency characteristic $\varphi_{OUT\text{-}IN}(f)$ [deg],
   – attenuation characteristic $k(f)$ [dB].
   Each plot must be accompanied by a sequence number and information about what is shown on the plot.
   ATENTION: Due to the wide frequency range, the logarithmic scale on the f [Hz] axis must be used on all plots.
   7.2. For the low-pass RC filter and high-pass RC filter:
      7.2.1. Read the cut-off frequency f_c from the plot for which the condition $U_{OUT}/U_{IN} = 1/\sqrt{2}$ is met, which corresponds to the so-called 3 decibel cut-off frequency, and estimate its maximum uncertainty Δf_c.
      ATTENTION: the uncertainty of the frequency measured with the generator is very small and has a negligible effect on the estimation of Δf_c. The maximum uncertainty Δf_c results mainly from the uncertainty $\Delta(U_{OUT}/U_{IN})$ of the voltage ratio U_{OUT}/U_{IN}.
      7.2.2. Calculate the theoretical cut-off frequency f_c according to formula (20) using the RC parameters given in Table 2.
      7.2.3. Compare the results obtained in points 7.2.1 and 7.2.2.
   7.3. For the Wien RC filter:
      7.3.1. Read the following values from the plot: the maximum $(U_{OUT}/U_{IN})_{max}$, the frequency f_0 corresponding to this maximum, and two cut-off frequencies f_{c1} i f_{c2} for 3 dB attenuation relative to the maximum of transmission (see Fig. 9). Use these values to calculate the Q factor according to the formula (30).
      7.3.2. Calculate the theoretical values of f_0 according to formula (28), $(U_{OUT}/U_{IN})_{max}$ according to formula (29) and the Q factor (31) using the RC parameters given in Table 2.
      7.3.3. Compare the results obtained in points 7.3.1 and 7.3.2.
   7.4. For the low-pass LC filter:
      7.4.1. Read the maximum $(U_{OUT}/U_{IN})_{max}$ from the plot and use this result to calculate the experimental value of the Q factor according to the formula (47).

7.4.2. Calculate the theoretical value of the Q factor (38) using the LC parameters given in Table 3. The value of the series resistance of the capacitor R_{C2} is not known, however, it can be assumed that $R_{C2} << R_{L1}$, which means that the approximate value of the series resistance $R \approx R_{L1}$.

7.4.3. Compare the results obtained in points 7.4.1 and 7.4.2.

7.5. For the high-pass LC filter (*extended version*)

7.5.1. Using the parameters of the LC elements given in Table 3, calculate the theoretical values of the asymptote intersection frequency: f_0 according to the formula (36) and f_p according to the formula (51).

7.5.2. Mark the frequencies f_0 and f_p calculated in point 7.5.1 on the $k(f)$ [dB] plot and draw three theoretical asymptotes according to the equations given below Fig. 18 on page 17.

7.5.3. Assess the compliance of theoretical asymptotes with the experimentally obtained dependence $k(f)$ [dB].

7.6. For the Wien LC filter (*extended version*)

7.6.1. Using the parameters of the LC elements given in Table 3, calculate the theoretical value of the frequency f_0 given by the formula (55) and mark it on the $U_{OUT}/U_{IN}(f)$ plot.

7.6.2. Evaluate the number of maxima on the $U_{OUT}/U_{IN}(f)$ plot occurring for a given position of the S2 switch in the experimental module.

8. Remarks and final conclusions.
Conclusions should contain observations on the course of the whole exercise. It is also necessary to summarize the result of comparison between of the experimental results and theoretical predictions, and to indicate particularly large discrepancies.

The report will be subject to the assessment of the presence and accuracy of all of the above components, clarity of presentation of the results (in the form of tables, graphs and results of calculations given together with appropriate descriptions and units of measure) and the quality of discussions and proposals formulated. Theoretical introduction is not required and is not included in the assessment.

Table 2. Averaged parameters of elements in RC filters for F1-01 ÷ F1-04 modules.

Position of the switch S1	R_1 [kΩ]	R_2 [kΩ]	C_1 [nF] for 1000 Hz	C_2 [nF] for 1000 Hz
1	0.498 ± 0.008	1.017 ± 0.020	213 ± 4	215 ± 6
2	0.498 ± 0.008	1.017 ± 0.020	456 ± 12	464 ± 11
3	0.498 ± 0.008	1.017 ± 0.020	967 ± 25	985 ± 15

Table 3. Averaged parameters of elements in LC filters for F1-01 ÷ F1-04 modules.

Position of the switch S2	L_1 [mH]	R_{L1} [Ω]	L_2 [mH]	R_{L2} [Ω]	C_1 [nF] for 1000 Hz	C_2 [nF] for 1000 Hz
1	3.9 ± 0.2	41.4 ± 2.3	1.00 ± 0.05	24.4 ± 0.7	218 ± 4	981 ± 23
2	3.9 ± 0.2	41.4 ± 2.3	3.9 ± 0.2	42.2 ± 1.9	467 ± 4	462 ± 11
3	3.9 ± 0.2	41.4 ± 2.3	33 ± 1	63.9 ± 2.3	1001 ± 49	217 ± 5

7. References

7.1. Basic reference materials

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7.2. Other reference materials

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