Exercise 321
Analysis of resonance in an RLC circuit

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The aim of this experiment is to familiarise with the RLC circuit and with the phenomenon of electric resonance. The task is to determine the resonance curve and its properties and to measure the unknown inductance of a coil.

1 Theoretical prerequisites

1. Harmonic oscillations (simple, dumped, driven). Resonance in oscillating system. Q-factor.

2. RLC circuit. Electric resonance. Complex impedance, resistance, reactance. Derivation of equation for resonance frequency.

3. Inductance of coils.

2 Equipment

Generator, mili-ammeter, decimal resistor, decimal condenser, decimal inductor, set of unknown coils.

3 Measurement method

Resonance in an RLC circuit is analysed by measurement of the current flowing in a circuit consisting of a serial connection of resistor, capacitor and coil, as a function of the generator frequency. The circuit is presented in Fig. 1. The magnitude of its total impedance is equal to
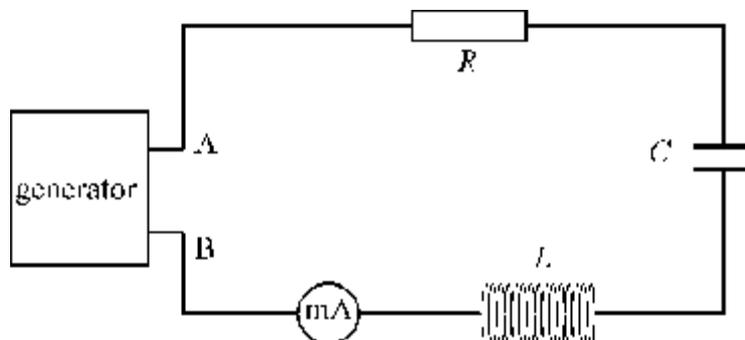

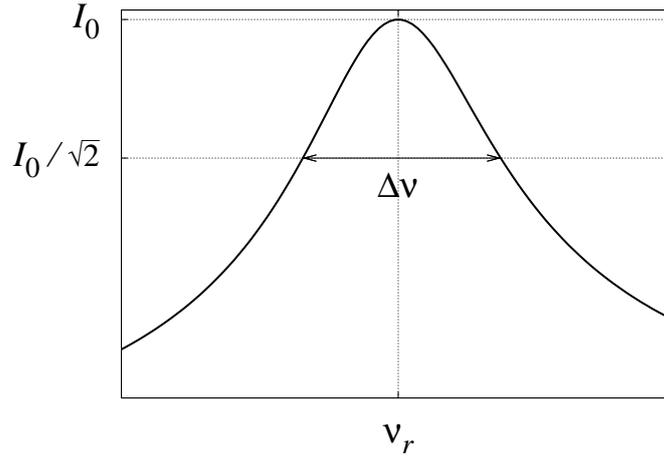Figure 1: Diagram of the electric set-up

Figure 2: Resonance curve

$$Z = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}, \tag{1}$$

where $\omega = 2\pi\nu$ is the angular frequency of the generator. Z has a minimal value—equal to R—if the resonance condition is fulfilled

$$L\omega = \frac{1}{C\omega}. \tag{2}$$

In such a case the electric current is maximum and equal to $I_0 = U_0/R$, where U_0 is the effective voltage of the generator. The frequency ν at which the above condition holds is called a resonance frequency ν_r (*what it is equal to?*). It can be determined from the equation

$$\nu_r = \frac{1}{2\pi\sqrt{LC}}. \tag{3}$$

For generator frequencies different from ν_r the current intensity in the circuit is smaller. The relation $I(\nu)$ is called a resonance curve and is depicted in Fig. 2. Its important parameter is a bandwidth (sometimes also called full-width at half-power) $\Delta\nu$. It is a width of the resonance curve at the level of the half of the maximum power P_0 stored in the system. As $P \propto I^2$ (*why?*), the bandwidth has to be measured for the current level $I_{1/2} = I_0/\sqrt{2}$.

The energy is dissipated by the emission of Joule's heat at the resistor. The amount of the energy loss in each cycle is characterised by a Q-factor, which is a product of an angular oscillator frequency and a time after which the stored energy decreases e times. It can also be determined and a ratio between the coil or capacitor resonant voltage to the generator voltage. Hence it is equal to

$$Q = \frac{L\omega_r}{R} = \frac{1}{\omega_r RC} = \frac{1}{R}\sqrt{\frac{L}{C}}. \tag{4}$$

There is a relation between the bandwidth and the Q-factor of the oscillator:

$$Q = \frac{\nu_r}{\Delta\nu} \tag{5}$$

4 Measurements and results analysis

Experimental set-up consists of an adjustable AC generator, decimal resistor, inductor and capacitor, and mili-ammeter connected in serial. The resonance curve is determined by reading the

current intensity in the circuit I as a function of generator frequency ν. The resonance frequency ν_r corresponds to the maximum current I_0.

The inductance of an unknown coil can be determined by measuring the resonance frequency of a circuit containing this coil. The relation between ν_r and L can be derived from Eq. (3).

4.1 Important steps

1. Connect the circuit according to the diagram from Fig. 1.

2. Set the elements to the following parameters: $R = 1\,\text{k}\Omega$, $C = 0.008\,\text{µF}$, $L = 0.07\,\text{H}$. Note down the settings and their uncertainties.

3. Set the generator range to $1\,\text{kHz}$, and mili-ammeter to $1\,\text{mA}$ AC. After verification of your connection by the teacher, turn on the generator.

4. Roughly find the generator frequency corresponding to the maximum current. Then adjust the generator gain to obtain the maximum current a little below $1\,\text{mA}$.

5. Set the frequency to $1\,\text{kHz}$ and read the current intensity. Increase the frequency by $0.1\,\text{kHz}$ and read the corresponding current intensity. Repeat this procedure until the frequency of $10\,\text{kHz}$. Note your results in the prepared table in your laboratory book (*what columns should this table contain?*).

6. Measure the inductance of an unknown coils: Set the capacitance of the condenser to $C = 0.005\,\text{µF}$. Then replace the decimal inductor with the set of unknown elements L_1–L_5 (discuss with your teacher which coils should be measured). Find the resonance frequency of the circuit corresponding to the maximum current. Repeat measurements several times for each coil. Note your results in the prepared table in your laboratory book (*what columns should this table contain?*).

4.2 Data analysis

When analysing your results you are supposed to:

1. Draw the resonance curve (remember not to directly connect the data points, but to approximate them with a smooth curve, e. g. 4th order polynomial). Determine the resonance frequency (in which the current reaches its maximum) and the bandwidth[1]. Compute the Q-factor (from Eq. (5)).

2. Compute the expected ν_r and the Q-factor using Eqs. (3) and (4). Compare the results with the ones obtained in point 1.

3. In both above cases determine the error (choose the proper method of error estimation).

4. Compute the inductance of the unknown coils using Eq. (3). Determine the error by the error propagation and statistical analysis of repeated measurements (using Student's distribution and confidence interval 90%). Decide what method of the error estimation is most appropriate in this case.

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[1]To improve precision you can find the maximum and points at which $I = I_{\text{max}}/\sqrt{2}$ numerically, using the polynomial approximation.