



Lodz University of Technology  
Institute of Physics

Laboratory of electronics

## Exercise E03IFE

Characteristics of Bipolar Junction Transistor (BJT)  
of n-p-n type in common-emitter circuit and  
characteristics of Field-Effect Transistor (FET) with  
n type channel

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Before starting the experiment one should learn the following theoretical subjects:

1. Conduction in semiconductors. [1], [2], [3], [4].
2. The p-n junction. The semiconductor diode – how does it work. [1], [2], [3].
3. The principle of operation of the bipolar junction transistor. [1], [2], [3].
4. The principle of operation of the field-effect transistor. [1], [2], [3].

## 1. Purpose of the exercise

The purpose of experiment is to gain knowledge on the static properties of bipolar transistor and field-effect transistor.

## 2. Hazards

Type	Absence	Low	Medium	High
electrical shock hazards		+		
optical radiation hazards	+			
mechanical hazards (including acoustic hazards, noise)	+			
electromagnetic radiation hazards (invisible)	+			
biological hazards	+			
ionizing radiation hazards	+			
chemical hazards	+			
thermal hazards (including explosion and fire)	+			

**The cables with banana-plugs are designed exclusively for use in low-voltage circuits – do not connect them to the mains supply 230 V.**

## 3. Introduction

### 3.1. Method of measurements of static properties of bipolar junction transistor

The bipolar junction transistor is investigated in common-emitter circuit. The power supply SPD3303D together with adjustable voltage regulator enable to apply voltage across collector-emitter junction of the transistor. The adjustable current source provides  $I_B$  current to base terminal of transistor. The kit of digital multimeters (DMMs) consists of two ammeters and two voltmeters which enable to measure: base current  $I_B$ , collector current  $I_C$ , collector-emitter voltage  $U_{CE}$  and base-emitter voltage  $U_{BE}$ . The obtained results allow to plot input, transfer, reversal and output characteristics of transistor and enable to calculate the h-parameters. In particular:

- the family of output characteristics –  $I_C = f(U_{CE}, I_B)$ , where  $I_B = \text{const.}$ ,
- the family of reversal characteristics –  $U_{BE} = m(U_{CE}, I_B)$ , where  $I_B = \text{const.}$ ,
- the family of input characteristics –  $U_{BE} = g(I_B, U_{CE})$ , where  $U_{CE} = \text{const.}$ ,
- the family of transfer characteristics –  $I_C = k(I_B, U_{CE})$ , where  $U_{CE} = \text{const.}$

### 3.2. Method of measurements of static properties of field-effect transistor

Field-effect transistor (FET) with n-type channel is investigated in the circuit with common source. The power supply SPD3303D together with adjustable voltage regulator enable to apply chosen  $U_{DS}$  voltage across drain-source junction. The adjustable voltage source provides  $U_{GS}$  voltage across the gate-source terminals. The kit of DMMs consists of an ammeter and voltmeters and enables to measure drain current  $I_D$  and  $U_{DS}$  and  $U_{GS}$  voltages. The obtained results allow to plot the transfer and output characteristics of transistor. In particular:

- the family of transfer characteristics –  $I_D = g(U_{GS}, U_{DS})$ , where  $U_{DS} = \text{const.}$ ,
- the family of output characteristics –  $I_D = g(U_{DS}, U_{GS})$ , where  $U_{GS} = \text{const.}$

## 4. Available equipment

### 4.1. Experimental module

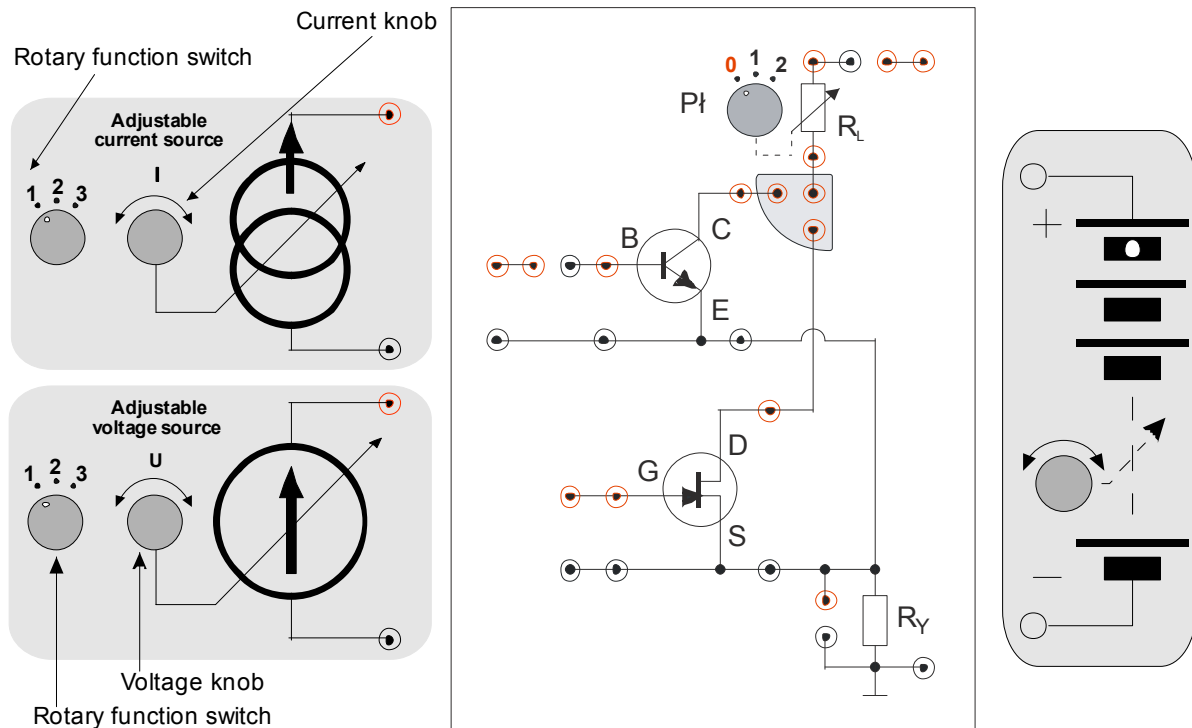


Fig. 1. The front panel of the experimental module (in the middle) with bipolar and field-effect transistors together with cooperating modules: the module ZŽS with adjustable current and voltage sources (left) and the module ZN with adjustable voltage regulator (right).

## **4.2. Multimeters**

Measurements of voltages and currents in the experimental module are performed using digital multimeters, AX-582B, KT890, M-3800, Protek 506 or UT804 (only selected models are available on a given measurement stand) [6]. Before measurements, check whether the proper measuring modes are selected with the switches of each DMM and whether the test probes are connected to the appropriate input terminals. Two of multimeters should play the role of ammeters and two other the role of voltmeters. One should also remember to choose the proper ranges of the DMMs.

## **4.3. Power supply**

The experimental module should be supplied only from ZŽS and ZN modules (Fig. 1). Before starting the measurements the +5 V output of stabilized power supply SPD3303D [6] should be connected with of the module ZŽS, and the CH1 output should be connected with the adjustable voltage regulator ZN. The voltage +25 V should be chosen for the CH1 output.

## **4.4. Oscilloscope**

Dynamic output characteristic for field-effect transistor can be obtained by use of digital oscilloscope SDS1052DL operating in XY mode [5,6]. After consulting the supervisor, the analogue oscilloscope GOS-620 or GOS-630 [5,6] can be used.

## **4.5. Function generator**

Function generator DF1641B [6] used to obtain the dynamic output characteristics of a field effect transistor.

## 5. Experimental procedure

The as reliable as possible measurements of the characteristics of transistors should be performed in order to obtain tens of measurement points.

### 5.1. The output and reversal characteristics of bipolar junction transistor without load

1. Connect the adjustable voltage regulator (ZN), adjustable DC current source (CS), milliammeter (mA), microammeter ( $\mu\text{A}$ ), voltmeters (V1 and V2) and jumpers according to the schematic diagram given in Fig. 2.

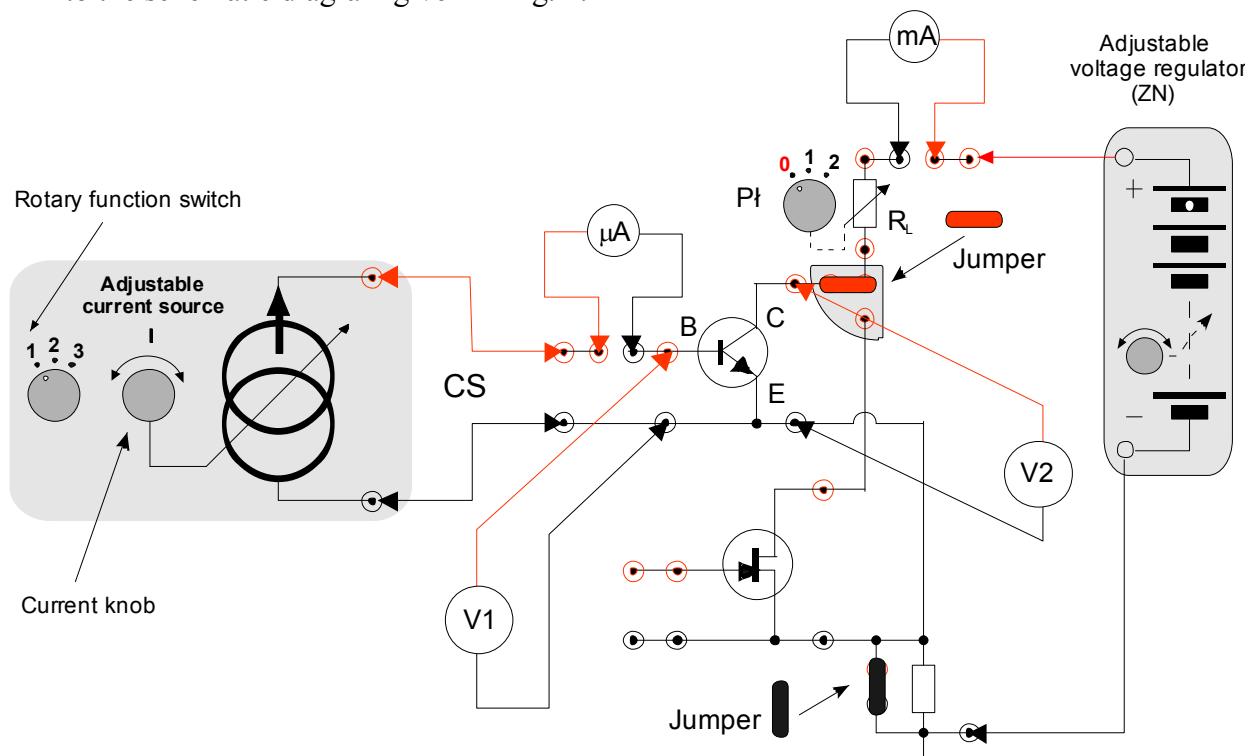


Fig. 2. Scheme of connections for bipolar junction transistor's (BJT) characteristics.

2. Set the rotary function switch on the adjustable current source (CS) to the „1” position (range from 0 to 0.5 mA).
3. Turn the current knob of the adjustable current source (CS) to the extreme left position.
4. Turn the voltage knob of the adjustable voltage regulator (ZN) to the extreme left position.
5. Select the 200 mA or 400 mA range of the DMM milliammeter (for measurement of  $I_C$ ).
6. Select the 200  $\mu\text{A}$  or 400  $\mu\text{A}$  range of the DMM microammeter (for measurement of  $I_B$ ).
7. Select the 20 V or 40 V range of the DMM voltmeter (for measurement of  $U_{CE}$ ).
8. Select the 2 V or 4 V range of the DMM voltmeter (for measurement of  $U_{BE}$ ).
9. Set the switch „Pl” in the experimental module to the „0” position. It means that  $R_L = 0 \Omega$ .
10. After supervisor's approval switch on the DMMs and DC power supply SPD3303D.  
**Warning! Do not change the position of voltage knob on DC power supply SPD3303D, which should provide a voltage of +25 V for the ZN module.**

11. Increase the voltage  $U_{CE}$  from 0 V to 10 V and determine the dependence of the collector current  $I_C$  and the  $U_{BE}$  voltage as a functions of the  $U_{CE}$  voltage at a given minimum current base  $I_B \approx 0$  (for such base current one should expected that values of collector current  $I_C$  will be close to zero).
12. Repeat the procedure described in previous point for at least two different current  $I_B$  values selected in the range from 40  $\mu\text{A}$  to 160  $\mu\text{A}$ . As many values of voltage  $U_{CE}$  as possible should be chosen from the range between 0 V and approximately 0.2 V (minimum 10 measuring points). To do this, rotate the voltage setting knob in the ZN module by a small angle each time.
13. Record the obtained results in Table 1.
14. Turn the current knob of the adjustable current source (CS) to the extreme left position.
15. Turn the voltage knob of the adjustable voltage regulator (ZN) to the extreme left position.
16. Plot the output characteristic i.e. the  $I_C(U_{CE})$  dependence for given value of the current base  $I_B$  and the reversal characteristic i.e. the  $U_{BE}(U_{CE})$  dependence for given value of the current base  $I_B$  (the plots can be made during writing report). As an example the family of  $I_C$  and  $U_{BE}$  versus  $U_{CE}$  curves for several values of  $I_B$  is presented in Fig. 3.

$U_{CE}$ [V]	$\Delta U_{CE}$ [V]	$I_B$ [ $\mu\text{A}$ ]	$\Delta I_B$ [ $\mu\text{A}$ ]	$U_{BE}$ [V]	$\Delta U_{BE}$ [V]	$I_C$ [mA]	$\Delta I_C$ [mA]

Table 1. Exemplary table for recording the results of U and I measurements along with their maximum uncertainties  $\Delta U$  and  $\Delta I$ , for the characteristics of bipolar transistor.

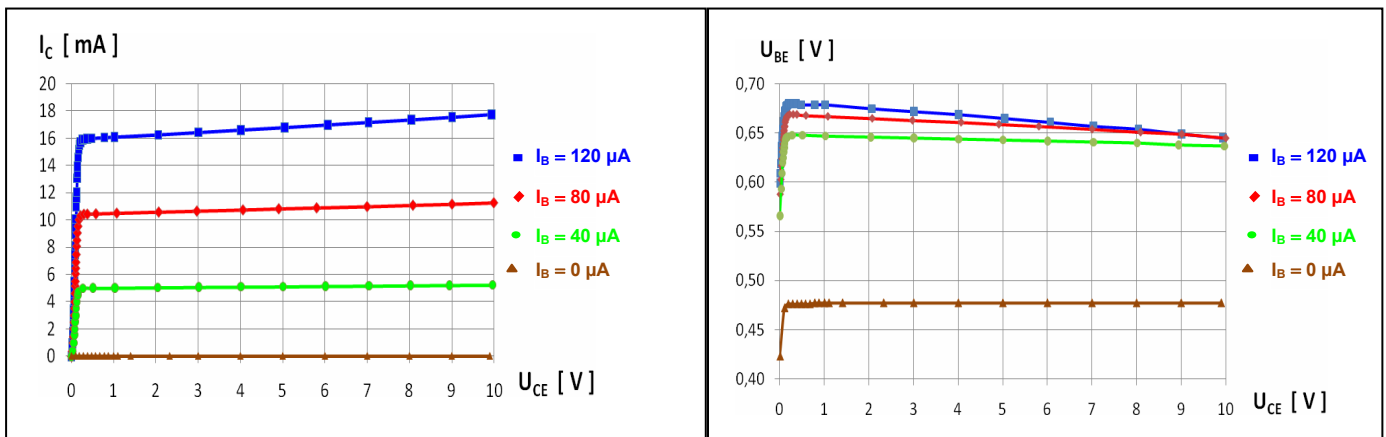


Fig. 3. An exemplary family the output and reversal characteristics for bipolar junction transistor without load.

## 5.2. The input and transfer characteristics of bipolar junction transistor

1. Turn the current knob on the adjustable current source (CS) to the extreme left position.
2. Set the switch „PI” in the experimental module to the „0” position. This means that  $R_L = 0 \Omega$ .
3. Set the voltage  $U_{CE}$  at value chosen from the range 2 V ÷ 10 V (ask the supervisor).
4. Increase the current  $I_B$  from 0  $\mu\text{A}$  to 160  $\mu\text{A}$  with step of 10  $\mu\text{A}$  and determine the dependence of  $U_{BE}$  on  $I_B$  and the dependence of  $I_C$  on  $I_B$ . Record the result in Table 1.

5. Repeat the procedure described in previous step setting another value of  $U_{CE}$  (ask supervisor).
6. Turn the current knob on the adjustable current source (CS) to the extreme left position.
7. Turn off the power supply.
8. Disconnect the adjustable power supply voltage (ZN), DC current source (CS), milliammeter (mA), microammeter ( $\mu\text{A}$ ), voltmeters (V1 and V2) and jumpers from the circuit shown in Fig. 2.
9. Plot the input characteristic i.e. the  $U_{BE}(I_B)$  dependence and the transfer characteristic  $I_C(I_B)$  for given values of  $U_{CE}$  (this can be done during writing report). As an example, the family of  $U_{BE}$  versus  $I_B$  and  $I_C$  versus  $I_B$  curves for two chosen voltages  $U_{CE}$  is presented in Fig. 4.

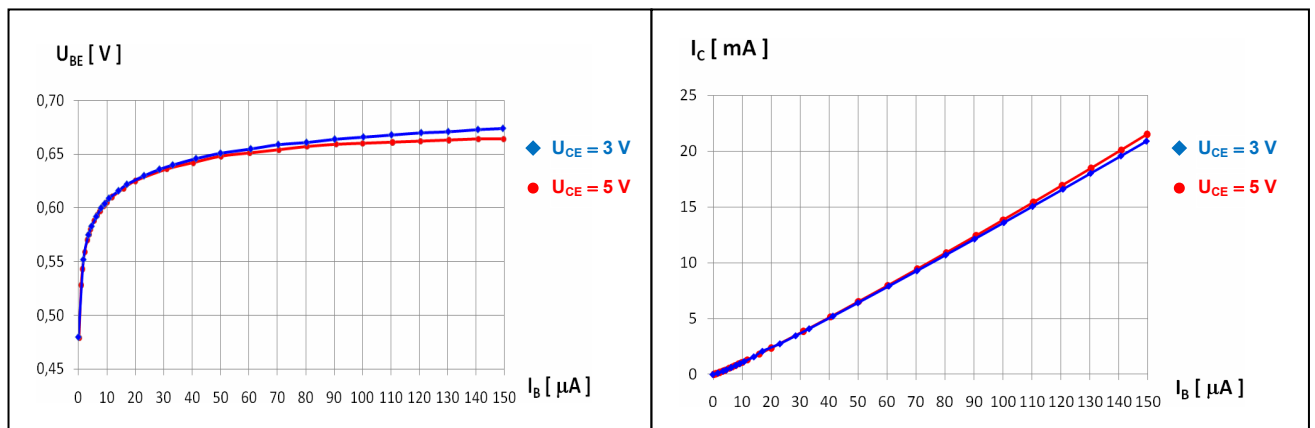


Fig. 4. An exemplary family of the input and transfer characteristics for bipolar junction transistor.

### 5.3. The transfer (gate) characteristic of field-effect transistor

1. Connect the adjustable voltage regulator (ZN), DC voltage source (VS), milliammeter (mA), voltmeters (V1 and V2) and jumpers according to the scheme given in Fig. 5.
2. Turn the voltage knob on the adjustable voltage regulator (ZN) to the extreme left position.
3. Set the rotary function switch on the adjustable voltage source (VS) to the „1” position (range from  $-12 V$  to  $0 V$ ).
4. Turn the voltage knob on the adjustable voltage source (VS) to the extreme left position.
5. Select the 200 mA or 400 mA range of the DMM milliammeter (for measurement of  $I_D$ ).
6. Select the 20 V or 40 V range of the DMM voltmeters (measurement of  $U_{GS}$  and  $U_{DS}$ ).
7. Set the switch „PF” in the experimental module to the „0” position. This means that  $R_L = 0 \Omega$ .
8. After supervisor’s approval, switch on the DMMs and DC power supply. **Warning! Do not change the position of the voltage knob on DC power supply SPD3303D.**
9. According to the supervisor’s order set the  $U_{DS}$  voltage (typically in the range from  $1 V$  to  $10 V$ ). Increasing the  $U_{GS}$  voltage from  $-10 V$  to  $0 V$  determine the  $I_D(U_{GS})$  dependence for fixed  $U_{DS}$ .
10. Repeat the procedure described in previous step setting the voltage  $U_{DS}$  at chosen values (ask supervisor).
11. Record the obtained results in Table 2.



12. Plot the  $I_D(U_{GS})$  dependences for chosen values of  $U_{DS}$  (this can be done during writing the report). As an example, the family of  $I_D$  versus  $U_{GS}$  curves for two chosen values of  $U_{DS}$  is presented in Fig. 6.
13. Determine the cut-off voltage  $U_{GS\ OFF}$  of FET transistor under the investigation.

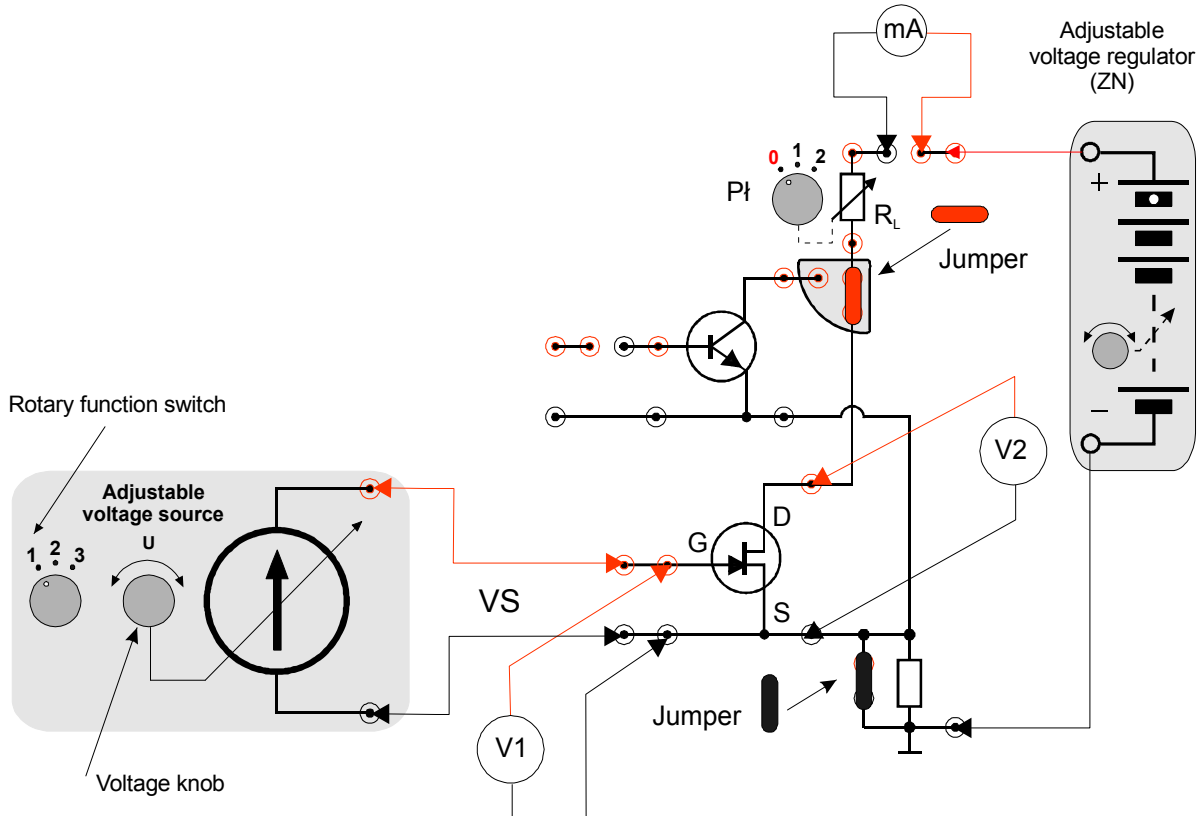


Fig. 5. Diagram of connections for FET transistor's characteristics.

$U_{GS}$ [V]	$\Delta U_{GS}$ [V]	$I_D$ [mA]	$\Delta I_D$ [mA]	$U_{DS}$ [V]	$\Delta U_{DS}$ [V]

Table 2. Exemplary table for recording the results of U and I measurements along with their maximum uncertainties  $\Delta U$  and  $\Delta I$ , for the characteristics of field-effect transistor.

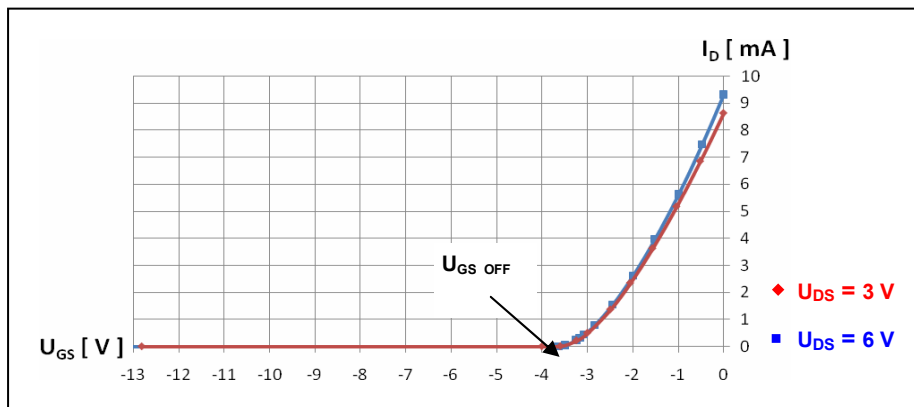


Fig. 6. An exemplary family of the transfer (gate) characteristics for field-effect transistor.

## 5.4. The output (drain) characteristic of field-effect transistor

1. Turn the voltage knob on the adjustable voltage regulator (ZN) to the extreme left position.
2. Set the rotary function switch on the adjustable voltage source (VS) to the „1” position (range from  $-12\text{ V}$  to  $0\text{ V}$ ).
3. Adjust the voltage source (VS) at  $U_{GS} \cong U_{GS\text{ OFF}}$
4. Select the 200 mA or 400 mA range of the DMM milliammeter (measurement of  $I_D$ ).
5. Set the switch „PF” in the experimental module to the „0” position. It means the  $R_L = 0\ \Omega$ .
6. Increase the  $U_{DS}$  voltage from  $0\text{ V}$  to  $10\text{ V}$  and determine the  $I_D(U_{DS})$  dependence for fixed  $U_{GS} \cong U_{GS\text{ OFF}}$ . As many values of voltage  $U_{DS}$  as possible should be chosen from the range between  $0\text{ V}$  and approximately  $4\text{ V}$  (minimum 10 measuring points). To do this, rotate the voltage setting knob in ZN by a small angle each time.
7. Repeat the procedure described in previous step setting the voltage  $U_{GS}$  at chosen values in range from  $U_{GS\text{ OFF}}$  to  $0\text{ V}$  (ask supervisor).
8. Record the obtained results in Table 2.
9. Plot the  $I_D(U_{DS})$  dependences for chosen values of  $U_{GS}$  (this can be done during writing the report). As an example, the family of  $I_D$  versus  $U_{DS}$  curves for three chosen values of  $U_{GS}$  is presented in Fig. 7.
10. Turn off the power supply.

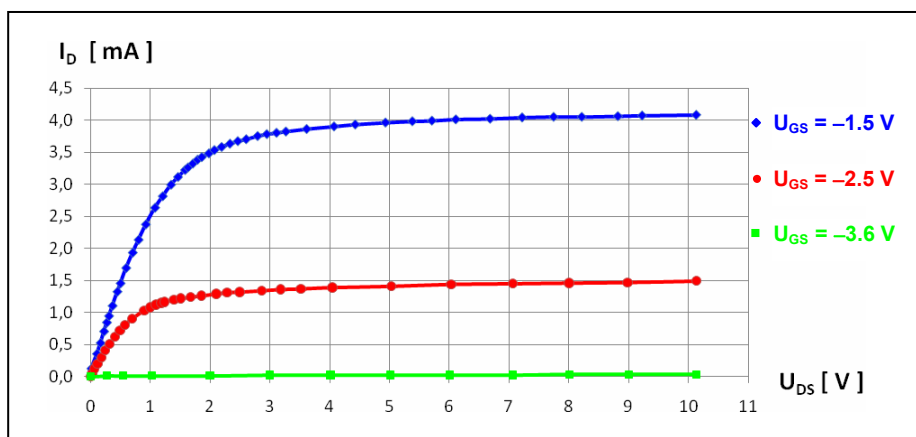


Fig. 7. Example of family of  $I_D$  versus  $U_{DS}$  curves for several values of  $U_{GS}$ .

## 5.5. Dynamic output characteristic of field-effect transistor (extended version)

1. Remove all jumpers from measurement panel.
2. Check out connections according to the scheme given in Fig. 8.
3. Connect the function generator output with CH1 input of oscilloscope ( $U_X$  in Fig. 8) and with D and S terminals of FET transistor using the T-type connector. Use BNC-BNC cable for connecting generator with oscilloscope and BNC–banana-plug cable for connecting generator with FET transistor. Pay attention to proper connections – black plug to the ground and the red plug to the D terminal of the FET transistor.
4. Connect the oscilloscope CH2 input with  $R_Y$  resistor ( $U_Y$  in Fig. 8) using the BNC–banana-plug cable. Pay attention to proper connections – black banana-plug connector to the ground and the red one to the S terminal of the FET transistor.

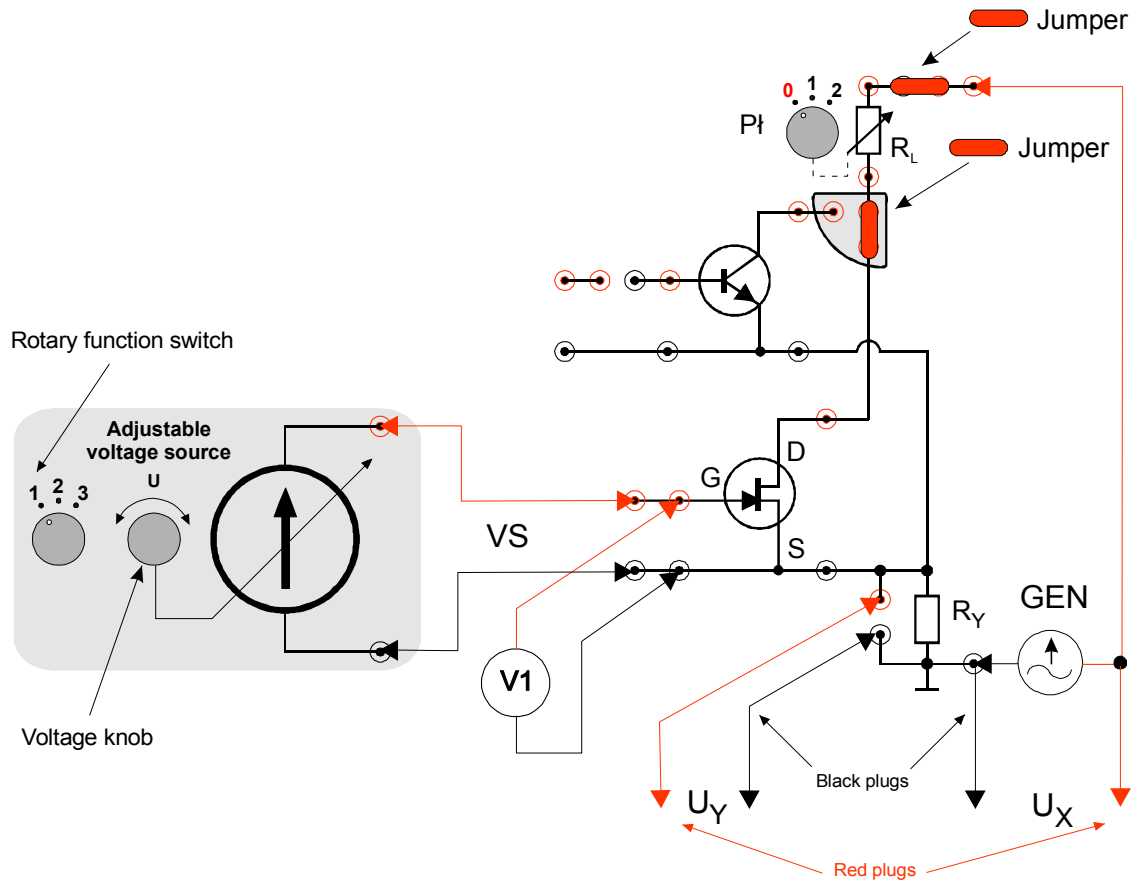


Fig. 8. Diagram of connections for observations of FET dynamic output characteristics.

5. After supervisor's approval, switch on the adjustable voltage source (VS), oscilloscope and function generator.
6. Adjust the voltage source (VS) at  $U_{GS} \cong U_{GS\ OFF}$ .
7. Select the triangle waveform by using the function select switch on the function generator front panel. Select the DC coupling mode on oscilloscope CH1 input. Set the output frequency on function generator to approx. 1 kHz and the output voltage signal to approx. 10 V (peak-to-peak). Activate DC OFFSET function on the front panel of generator and adjust the DC offset voltage (the triangle signal on the oscilloscope's screen must be shifted up by half of the peak to peak value  $U_{PP}$ ). The adjusted output signal of the generator should vary from 0 V to 10 V (it must be greater than zero volts).
8. Put the jumper in the "drain" circuit. Leave the  $R_Y$  resistor to be un-short-circuited.
9. Switch the oscilloscope into X-Y mode.
10. Change the  $U_{GS}$  voltage from  $U_{GS\ OFF}$  to 0 V for several chosen voltages of  $U_{GS}$ .
11. Write down the main settings of the oscilloscope and copy the signals from the oscilloscope's screen.
12. After disconnecting the circuit, measure the  $R_Y$  resistance.
13. Preparing the report, compare qualitatively and quantitatively the oscillograms obtained from the dynamic output characteristic with corresponding drain characteristics obtained using the "point-by-point" method.

## 6. Guidelines on writing reports

Report has to be composed of:

1. Front page (by using a template).
2. Description of experiment purposes.
3. Short introduction (basic definitions, formulas, description of used marks and symbols).
4. Schematic diagrams of tested circuits.

The report should contain only diagrams of the systems, which were actually compiled during the measurements. Each scheme must be accompanied by a sequence number and entitled. All the components shown in the diagram must be clearly described and identified using commonly used symbols.

5. List of used instruments and devices (id/stock number, type, setting and range values).
6. Results of measurements (including tables with estimation of maximum measurement uncertainty). Each table should have its number and title.
7. Plots, calculations, analysis, interpretation and sub-conclusions related to all required points of “Experimental procedure”. Each plot showing the results of measurements should have its number and title containing information about what a given plot presents.

7.1. For bipolar junction transistor (BJT):

- a) plot of the  $(I_C(U_{CE}) I_B=\text{const})$  dependences,
- b) plot of the  $(U_{BE}(U_{CE}) I_B=\text{const})$  dependences,
- c) plot of the  $(U_{BE}(I_B) U_{CE}=\text{const})$  dependences,
- d) plot of the  $(I_C(I_B) U_{CE}=\text{const})$  dependences.

7.2. According to the h-parameter model of bipolar junction transistor for a common emitter circuit the  $U_{BE}$  voltage and  $I_C$  current can be expressed as functions of  $I_B$  current and  $U_{CE}$  voltage:

$$U_{BE} = \left( \frac{\partial U_{BE}}{\partial I_B} \right)_{U_{CE}=\text{const}} \cdot I_B + \left( \frac{\partial U_{BE}}{\partial U_{CE}} \right)_{I_B=\text{const}} \cdot U_{CE} , \quad (1)$$

$$I_C = \left( \frac{\partial I_C}{\partial I_B} \right)_{U_{CE}=\text{const}} \cdot I_B + \left( \frac{\partial I_C}{\partial U_{CE}} \right)_{I_B=\text{const}} \cdot U_{CE} ,$$

where the partial derivatives of relevant functions define:

a) input resistance:

$$h_{11e} = \left( \frac{\partial U_{BE}}{\partial I_B} \right)_{U_{CE}=\text{const}} \quad (2)$$

(which varies from 100  $\Omega$  to 5 k $\Omega$ ),

b) DC reverse transfer voltage gain:

$$h_{12e} = \left( \frac{\partial U_{BE}}{\partial U_{CE}} \right)_{I_B=\text{const}} \quad (3)$$

(which varies from 0.01 to 0.0001),

c) DC current gain:

$$h_{21e} = \left( \frac{\partial I_C}{\partial I_B} \right)_{U_{CE}=\text{const}} \quad (4)$$

(which varies from 30 to 300),

d) output conductance:

$$h_{22e} = \left( \frac{\partial I_C}{\partial U_{CE}} \right)_{I_B = \text{const}} \quad (5)$$

(which varies from  $10^{-6}$  S to  $10^{-3}$  S; S – siemens).

Hence, the voltage  $U_{BE}$  and current  $I_C$  can be described as:

$$U_{BE} = h_{11e} \cdot I_B + h_{12e} \cdot U_{CE} \quad (6)$$

$$I_C = h_{21e} \cdot I_B + h_{22e} \cdot U_{CE} \quad (7)$$

To obtain the h-parameters, the relevant partial derivatives should be calculated and characteristics of BJT can be applied for this. All static characteristics are shown in Fig. 9. The yellow areas indicate the data ranges used for calculations.

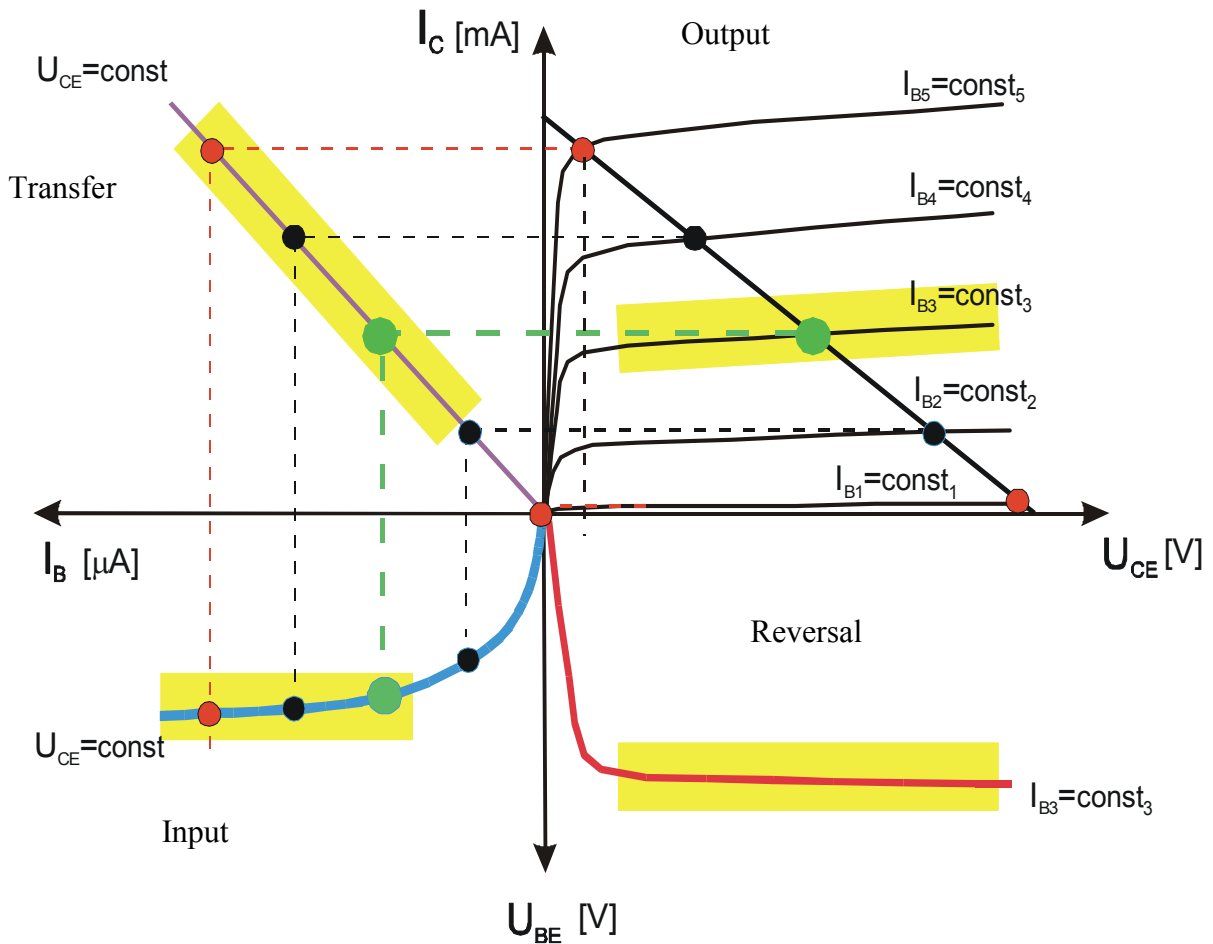


Fig. 9. Example of merged characteristic of BJT in common emitter circuit.

In practice, a set of input, reversal, transfer and output transistor characteristics is equivalent to the merged static characteristic. An example of a set of characteristics of the bipolar transistor for selected parameters  $I_B$  and  $U_{CE}$  is shown in Fig. 10.

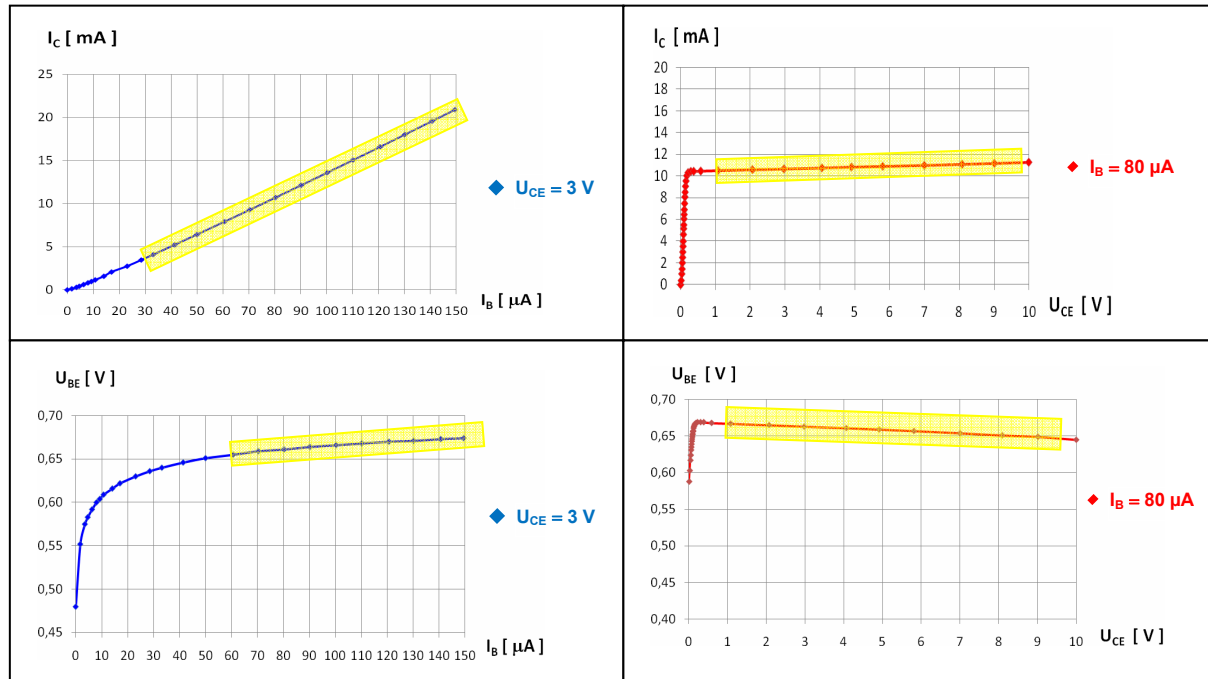


Fig. 10. Exemplary set of characteristics of bipolar transistor in common emitter circuit.

The yellow areas in Figs. 9 and 10 indicate the linear segments of characteristics, which should be used for calculation of dynamic hybrid coefficients, i.e. partial derivatives  $h_{11e}$ ,  $h_{12e}$ ,  $h_{21e}$  and  $h_{22e}$ , defined on pages 12 and 13).

7.3. Prepare the set of input, reversal, transfer and output characteristics of the **bipolar** transistor.

Basing on the linear segments of characteristics (yellow areas in Fig. 10) **calculate** the hybrid coefficients:

- DC current gain  $h_{21e}$  for chosen  $U_{CE}$  voltage,
- DC reverse transfer voltage gain  $h_{12e}$  for chosen base current  $I_B$ ,
- input resistance  $h_{11e}$  for chosen  $U_{CE}$  voltage,
- output conductance  $h_{22e}$  for chosen base current  $I_B$ .

**Calculations** together with **estimation of standard uncertainties** of obtained values should be performed with **least squares method**. Remarks on the least squares method can be found in section A1 of Appendix to this manual. Special attention should be paid to proper units of the calculated quantities.

7.4. Knowing the hybrid coefficients, one should present the following parameters of the bipolar transistor: **current gain** ( $h_{21e}$ ), **voltage gain** (reciprocal of reverse transfer voltage gain  $h_{12e}$ ), input resistance ( $h_{11e}$ ) and output resistance (reciprocal of output conductance  $h_{22e}$ ).

7.5. The results obtained for the **unipolar (field-effect) transistor** should be gathered in Table 2 and:

- prepare plot of the transfer characteristic  $I_D = f(U_{GS}, U_{DS})$  for chosen  $U_{DS}$  voltage,

- b) determine the cut off voltage  $U_{GS\ OFF}$ ,
- c) prepare plot of the output characteristic  $I_D = g(U_{DS}, U_{GS})$  for chosen  $U_{GS}$  voltage.

The set composed of transfer and output characteristics determined for the unipolar transistor can be presented in the same way as for the bipolar transistor. Example is shown in Fig. 11.

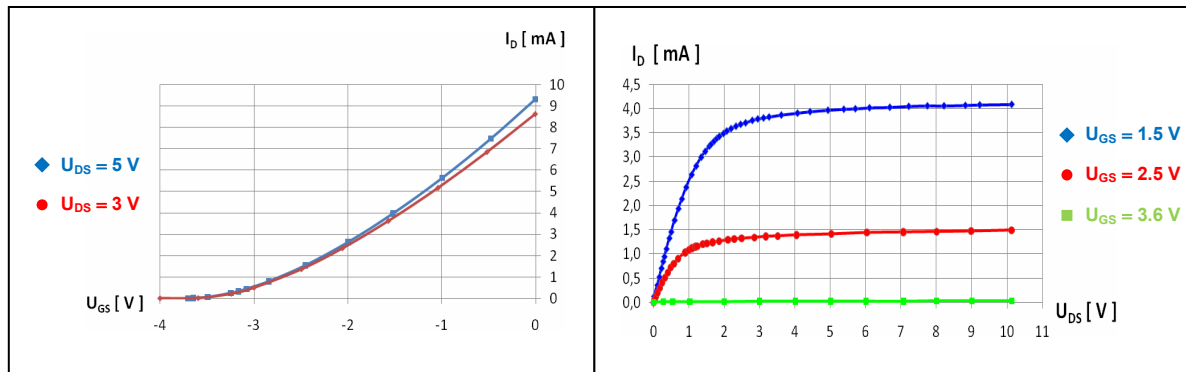


Fig. 11. Exemplary set of characteristics of the field-effect transistor.

- 7.6. Compare qualitatively and quantitatively the oscillograms obtained from the dynamic output characteristic of the field-effect transistor with corresponding characteristics obtained using the “point-by-point” method. All oscillograms must be clearly described including information on the gain coefficients of the oscilloscope for the X and Y inputs, the values of the  $U_{DS}$  voltage, and the  $R_Y$  resistance.
- 7.7. **Attention! Each plot as well as each result of calculations should be commented.** Final conclusions are also necessary. Conclusions should contain estimation of accuracy of the measurements and own remarks concerning the exercise.

#### 8. Remarks and final conclusions.

The comments should contain estimation of the accuracy of measurements and own observations about the course of the exercise.

The report will be subject to the assessment of the presence and accuracy of all of the above components, clarity of presentation of the results (in the form of tables, graphs) and the quality of discussions and proposals formulated. Theoretical introduction is not required and in case of its inclusion in the report it will not affect the assessment.

## 7. References

### 7.1. Basic reference materials

- [1] R. Śledziwski, *Elektronika dla Fizyków*, PWN, Warszawa, 1984.
- [2] K. Braclawski. A. Siennicki, *Elementy półprzewodnikowe*, WSiP, Warszawa, 1986.
- [3] A. Rusek, *Podstawy elektroniki – tom I i II*, WSiP, Warszawa, 1984.
- [4] A. Sukiennicki, A. Zagórski, *Fizyka ciała stałego*, WNT, Warszawa, 1984.
- [5] J. Rydzewski, *Pomiary oscyloskopowe*, WNT, Warszawa, 1994.

### 7.2. Other reference materials

- [6] User manuals of the power supply, DMMs, oscilloscopes and function generator  
<https://fizyka.p.lodz.pl/pl/dla-studentow/information-technology/fundamentals-of-electronics/>
- [7] B. Żółtowski, *Laboratory data analysis*, Skrypt PŁ, Łódź, 2005.  
– web page of Electronics Foundation (fizyka.p.lodz.pl).



## 8. Appendix

### A1. Least squares method

A complete description of the least squares method can be found in the script [7], which is available as a PDF file on the webpage with auxiliary materials for students taking classes in Laboratory of electronics. In this appendix, the scope of the presentation of the method corresponds to the scope of its use in Laboratory of electronics.

Experimentalists often need to find analytical form of the functional relation  $y = f(x)$  based on  $n$  pairs of measurement results ( $x_i$  and  $y_i$ , where  $i = 1, 2, \dots, n$ ), where the values of  $x$  and  $y$  change in some ranges. This process is called fitting, approximation or regression. In the case of experiments performed in Laboratory of electronics, the general form of the function results from known theoretical relationships or definitions of evaluated quantities and the purpose of regression is to determine the values of parameters in the assumed general form of the function. There are several methods used to solve this problem, among which the so-called **least squares method** is most often used. Assuming that each measurement result  $y_i$  is subject to Gaussian distribution with the same uncertainty and that the uncertainties of  $x_i$  are negligibly small, the method is based on minimizing the sum of squared differences between the actual values of the dependent variable  $y_i(x_i)$  and the values predicted by the model function  $f(x_i, a, b, c, \dots)$

$$S = \sum_{i=1}^n [y_i - f(x_i, a, b, c, \dots)]^2 . \quad (\text{A1.1})$$

The sum  $S$  reaches the minimum with respect to all the parameters of the function  $f$ , i.e.  $a, b, c, \dots$ , when  $\partial S / \partial a = 0$ ,  $\partial S / \partial b = 0$ ,  $\partial S / \partial c = 0$ , ... . An analytical solution of the system of equations obtained in this way is known only for some forms of the function  $f$ , however, minimization of the sum  $S$  for other functions can usually be done numerically. In the exercises available at Laboratory of electronics in Institute of Physics, only **linear regression** is used, where the goal is to adjust the coefficients of a linear function.

In the case of the linear function  $\mathbf{f(x)} = \mathbf{a x + b}$  the solution of the system of equations  $\partial S / \partial a = 0$  and  $\partial S / \partial b = 0$  leads to finding the coefficients given by the formulas:

$$a = \frac{n \sum_{i=1}^n x_i y_i - \sum_{i=1}^n x_i \cdot \sum_{i=1}^n y_i}{D} , \quad (\text{A1.2})$$

$$b = \frac{\sum_{i=1}^n x_i^2 \cdot \sum_{i=1}^n y_i - \sum_{i=1}^n x_i \cdot \sum_{i=1}^n x_i y_i}{D} , \quad (\text{A1.3})$$

where

$$D = n \sum_{i=1}^n x_i^2 - \left( \sum_{i=1}^n x_i \right)^2 . \quad (\text{A1.4})$$

The precision of the estimated coefficients a and b depends on their standard uncertainties of type A, i.e. the standard deviations u(a) and u(b) given by the formulas:

$$u(a) = \sqrt{\frac{n \sum_{i=1}^n (y_i - ax_i - b)^2}{(n-2)D}}, \quad (\text{A1.5})$$

$$u(b) = \sqrt{\frac{\sum_{i=1}^n x_i^2 \cdot \sum_{i=1}^n (y_i - ax_i - b)^2}{(n-2)D}}, \quad (\text{A1.6})$$

where D is defined by equation (A1.4). In addition, a statistical relationship (correlation) between the values of two variables  $x_i$  and  $y_i$  may be assessed by calculating the linear correlation coefficient defined as:

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \cdot \sum_{i=1}^n (y_i - \bar{y})^2}}. \quad (\text{A1.7})$$

The values of the r coefficient vary from  $-1$  to  $+1$ . The limit values  $r = \pm 1$  indicate an ideal linear correlation, while  $r = 0$  means no correlation and an attempt to fit a linear function is unjustified in this case. A negative sign of the correlation coefficient means that an increase in the value of one variable is associated with a decrease in the value of other variable, while a positive sign means the same direction of changes of both variables.

If a theoretical relationship or a definition of estimated quantity corresponds to the linear function with zero y-intercept,  $f(x) = a x$ , the slope should be calculated from the formula:

$$a = \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2}, \quad (\text{A1.8})$$

and its standard deviation according to the formula

$$u(a) = \sqrt{\frac{\sum_{i=1}^n (y_i - a x_i)^2}{(n-1) \sum_{i=1}^n x_i^2}}. \quad (\text{A1.9})$$

The author of the report should always consider which of the two cases discussed above is appropriate in the context under consideration.

In practice, statistical functions available in Microsoft Excel and Open Office Calc programs are often used instead of performing successive stages of calculations according to the formulas (A1.2)-(A1.9). The function called REGLINP is particularly powerful, and its syntax is as follows:

**=REGLINP(range\_y; range\_x; constant; stats)**

where

- range\_y – the range of cells containing  $y_i$  values,
- range\_x – the range of cells containing  $x_i$  values,
- constant – the logical value PRAWDA which allows to calculate the y-intercept b, or the value FAŁSZ which means  $b = 0$ . All numeric values other than zero, e.g. 7, are interpreted as PRAWDA, and 0 corresponds to FAŁSZ,
- stats – a logical value specifying whether to calculate additional regression statistics; if stats is FAŁSZ only the coefficients a and b are returned.

As the REGLINP function returns a two-dimensional array of values, it should be entered as an array formula. To do this, you need to first highlight the range of cells for the function result, type your formula in the edit bar above the sheet, and then press the key combination Ctrl + Shift + Enter to enter the formula into the sheet.

Assume that cells A1:A5 contain known  $x_i$  values, cells B1:B5 contain known  $y_i$  values, and the area C1:D5 is selected as in example Table A1.1 below. When you type the following formula in the edit bar

**=REGLINP(B1:B5; A1:A5; 1; 1)**

and press the Ctrl + Shift + Enter combination, the calculation results will appear in the previously selected area. Starting from the upper-left corner of the area highlighted in light blue in Table A1.1, you will see the results returned in the following order:

- a – the value of the slope calculated according to the formula (A1.2),
  - b – the value of the y-intercept – the formula (A1.3),
  - u(a) – the value of standard deviation for the coefficient a – the formula (A1.5),
  - u(b) – the value of standard deviation for the coefficient b – the formula (A1.6),
  - $r^2$  – the square of the correlation coefficient r calculated according to the formula (A1.7),
- and further results F,  $d_f$ ,  $SS_{reg}$ ,  $SS_{resid}$ , which do not apply in Laboratory of electronics.

	A	B	C	C	D
1	1	110		a	b
2	2	131		u(a)	u(b)
3	3	192		$r^2$	$se_y$
4	4	317		F	$d_f$
5	5	489		$SS_{reg}$	$SS_{resid}$

Table A1.1. Sample data for calculations ( $x_i$  in column A and  $y_i$  in column B) and the order of regression results returned by the REGLINP function in the area highlighted in light blue.

If the constant argument has the value FAŁSZ, the results are returned in the order given above, except that the coefficient b is zeroed, and the result u(b) is absent.

If an array formula is difficult to understand, it can be replaced by a set of scalar formulas with the INDEKS function. Calling the function with the following arguments

**=INDEKS(array; row\_number; column\_number)**

returns the value of a single cell from the argument array at a position given by (row\_num; column\_num) coordinates, which are counted from (1;1) in the upper-left corner of the array. Thus, a number of scalar formulas must be used to obtain a set of results equivalent to calling REGLINP as a single array formula. Alternatively, some of the results returned in the table by the REGLINP function can also be obtained using specialized scalar functions NACHYLENIE, ODCIĘTA and R.KWADRAT.

a:	=INDEKS(REGLINP(B1:B5; A1:A5; 1; 1); 1; 1)	or	=NACHYLENIE(B1:B5; A1:A5)
b:	=INDEKS(REGLINP(B1:B5; A1:A5; 1; 1); 1; 2)	or	=ODCIĘTA(B1:B5; A1:A5)
u(a):	=INDEKS(REGLINP(B1:B5; A1:A5; 1; 1); 2; 1)		
u(b):	=INDEKS(REGLINP(B1:B5; A1:A5; 1; 1); 2; 2)		
r <sup>2</sup> :	=INDEKS(REGLINP(B1:B5; A1:A5; 1; 1); 3; 1)	or	=R.KWADRAT(B1:B5; A1:A5)

Table A1.2. Scalar replacements for the REGLINP function entered as an array formula.

It should be remembered that the format of writing the result obtained together with its uncertainty informs about the type of this uncertainty. For example, the result  $a = 136.173 \Omega$  together with its standard uncertainty  $u(a) = 1.187 \Omega \approx 1.2 \Omega$  should be written as:

$$a = 136.2(1.2) \Omega \quad \text{the final result together with standard uncertainty}$$

If expanded uncertainty was calculated  $U$  – e.g. for the above result obtained for series of  $n = 22$  pairs of measurements  $U(a) = k(P, \nu) u(a) = 3.377 \Omega$ , where  $k(P, \nu) = 2.845$  is the coverage factor described by t-Student distribution for the assumed confidence level  $P = 99 \%$  and the number of degrees of freedom  $\nu = 20$  ( $\nu = n - 2$ , where 2 is the number of regression parameters) – please write:

$$a = (136.2 \pm 3.4) \Omega \quad \text{the final result together with expanded uncertainty}$$

**WARNING!** Depending on the character for the list separator selected in the regional settings of the operating system, **commas** or **semicolons** should be used in Microsoft Excel formulas. In the case of Open Office Calc, the list separator may be determined by the language version of the program.

The list separators and names of the functions and logical constants described above refer to Polish versions of programs. Users of other versions must use other language-specific list separators and names. In addition, the names have been translated only for selected languages. For all other languages, the English names are used. During the preparation of this appendix, this concerned, for example, the Ukrainian language.

Polish	English and Ukrainian
FAŁSZ	FALSE
INDEKS	INDEX
NACHYLENIE	SLOPE
ODCIĘTA	INTERCEPT
PRAWDA	TRUE
R.KWADRAT	RSQ
REGLINP	LINEST

Table A1.3. Names of selected functions and constants used in Polish, English and Ukrainian versions of spreadsheets.