Experimental Methods in Science

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Part I

Experimental Data Acquisition and Analysis

Experimental Data Acquisition and Analysis: Outline

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1 Experiments in Physics

Scientific method

Body of techniques for investigating phenomena, acquiring new knowledge, or correcting and integrating previous knowledge. Based on gathering observable, empirical and measurable evidence subject to specific principles of reasoning.

- **Characterisations** (observations, definitions, and measurements of the subject of enquiry)
- **Hypotheses** (theoretical, hypothetical explanations of observations and measurements of the subject)
- **Predictions** (reasoning including logical deduction from the hypothesis or theory)
- **Experiments** (tests of all of the above)

Experiment and measurement

Definition 1. Experiment is a set of observations performed in the context of solving a particular problem or question, to retain or falsify a hypothesis or research concerning phenomena. The experiment is a cornerstone in the empirical approach to acquiring deeper knowledge about the physical world.
ex- periri (la): of/from trying

Measurement is the estimation of the magnitude of some attribute of an object, such as its length or weight, relative to a unit of measurement. A measurement is understood to have three parts: 1) the measurement itself, 2) the margin of error, 3) the confidence level.

1.1 Experiment: Step-by-Step

Elements of experiment

1. Specification of the goal
2. Planning
3. Preparation
4. Draft measurements
5. Experimental data acquisition
6. Data analysis
7. Presentation of results

Specification of the goal

- Verification of the hypothesis
  - Foucault's pendulum
  - Young's light-interference experiment
  - Michelson-Morley experiment
- Measurement of some quantity
  - Millikan's oil-drop experiment
  - Eratosthenes's measurement of the Earth's circumference
  - Cavendish's torsion-bar measurement of the gravitational constant
- Design of a new experimental method
- Verification of the equipment applicability

Sometimes we observe unpredicted results (discovery of Roentgen's X-rays).

Planning

- Specify which properties of the analysed phenomenon are the most important for your goal and which quantities are interesting.
- Derive the necessary mathematical relations.
- Determine which quantities need to be directly measured and which need to be determined indirectly.
- Decide what should be the range of measurements and what is the required precision.
- Estimate possible sources of errors and find a way to eliminate them.
- Design the experimental set-up.

Preparation

- Collect equipment.
- Read manuals, learn equipment parameters.
- Construct an experimental set-up.
- Decide what to directly measure and in what order.
- Prepare your laboratory notebook for making notes (tables etc.).

Draft measurements

- Get used to your equipment.
- Check correctness of your procedure.
- Verify if there are no systematic errors.

- Make draft measurement of the sample with already known parameters.
- Make rough calculations and analyse your results.
- Check if your result is what you expect.

Experimental data acquisition

- Begin only after completing the previous steps.
- Don't rush!
- Note down every result and all the necessary additional information.
- Always note raw reads. You can make necessary calculations (e. g. scaling) in the mean time.
- Repeat your measurements several times.
- Don't forget about units!
- Always note the precision of the instruments.

Data analysis

- Make all the necessary calculations.
- Try to realise your original goal: verify hypothesis, give measured value etc.

Presentation of results

- Prepare report, publication, presentation, poster etc.
- Mind that your reader does not need to be familiar with your experiment.
- Be concise and to the point!

1.2 Data Acquisition and Making Notes

Making notes

Forms of laboratory notes

- Sheets of paper
- Old-style lab-book
- Laptop
- Computer designated for data acquisition

Complete: give all the information about the set-up and the experiment
Clear and legible: you should be able to use them after long time

Elements of the laboratory notes
Compulsory

- Information about experiment: title, date, who did it;
- Definitions of all the quantities and symbols;
- Information about the apparatus: type, settings, ranges, precisions etc.;
- Diagram of the set-up;
- Measured quantities.

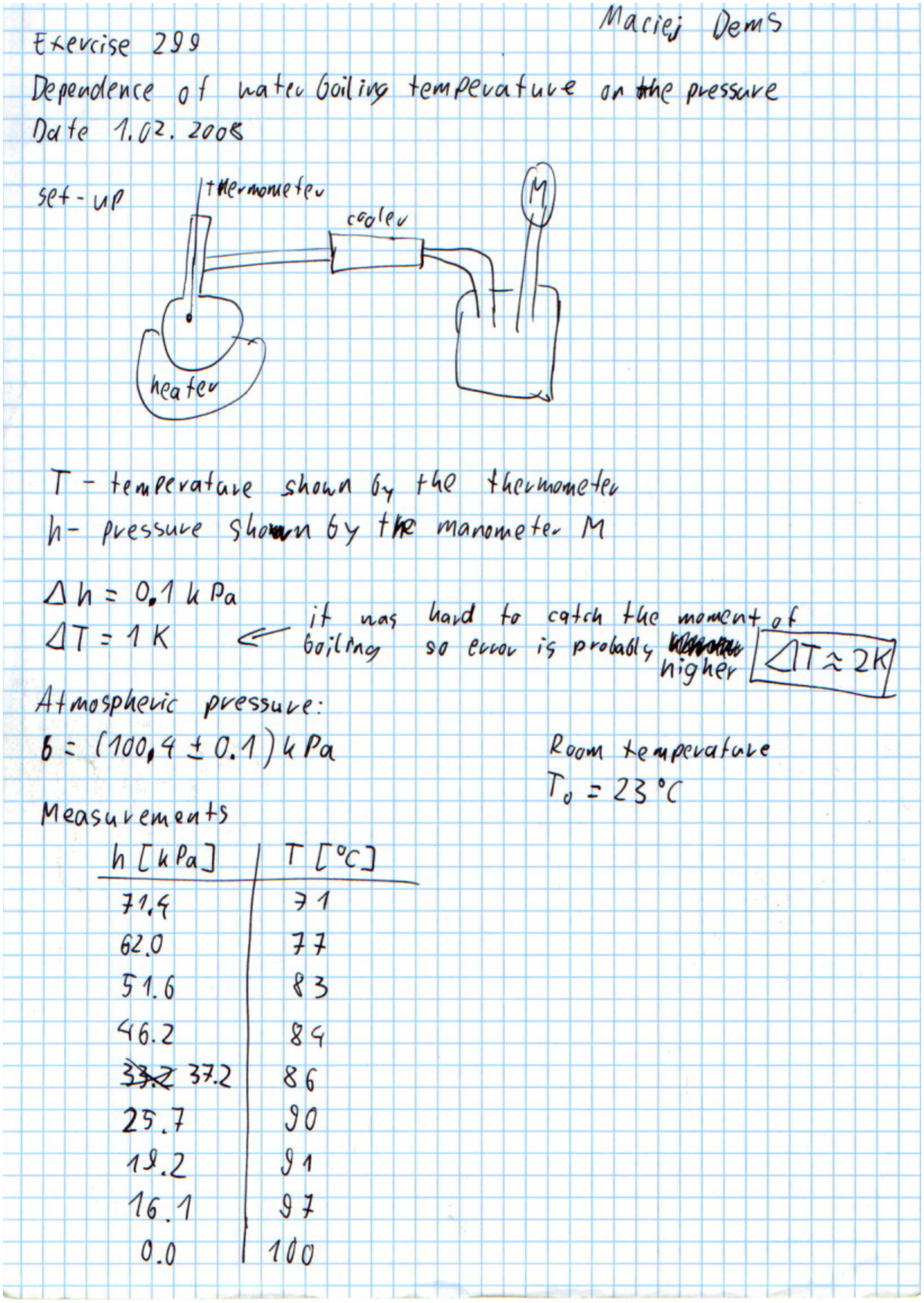
Optional but useful

- Information about conditions in the laboratory (temperature, humidity, atmospheric pressure, etc.)—especially when it can influence the results;
- Rough plots based on the draft measurements;
- All other comments.

Rules of making notes

- Don't make draft notes. Always keep your original notes and keep them tidy form the beginning.
- All the measured quantities should be put "raw" as read from the apparatus and as quick as possible. The necessary scaling should be done later.
- Always give information about the unit, either directly or indirectly by giving the range and scale for later processing.
- All the corrections must be clear. Use ~~45.3~~ 46.8 instead of 46.8.
- Put your results in tables where possible.
- For each quantity use exactly one unique symbol. Give its definition.
- Write down comments and clarifications. Your notes should be legible for someone else and for you after months.

Example of lab notes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Exercise 299

Dependence of water boiling temperature on the pressure

Date 1.02.2008

set-up

T – temperature shown by the thermometer

h – pressure shown by the manometer M

$\Delta h = 0.1$ kPa

$\Delta T = 1$ K ← it was hard to catch the moment of boiling so error is probably higher $\boxed{\Delta T \approx 2K}$

Atmospheric pressure:

$b = (100.4 \pm 0.1)$ kPa

Room temperature

$T_0 = 23\,°C$

Measurements

h [kPa]	T [°C]
71.4	71
62.0	77
51.6	83
46.2	84
~~33.2~~ 37.2	86
25.7	90
18.2	91
16.1	97
0.0	100

1.3 Notation of Measured Quantities

Two rules of thumb

Always give a unit!

A number without unit is useless! It is even less useful than the unit without a number.

Note the uncertainty of the number!

You cannot make exact measurements. Hence it is important to give the maximum error.

$$m = (123.456 \pm 0.015)\,\mathrm{kg}$$

Notation of approximate numbers

- Uncertainty is always rounded up (you cannot decrease uncertainty) and
  - Can have only one significant figure.
  - If the first significant figure is 1, then leave two significant figures.
- The same unit for the quantity and uncertainty.
- Quantity rounded to match decimal digits of uncertainty.

Definition 2. Significant figures: all digits in the number except leading zeros.
0.00**123**, **1.203**, 0.0**12300**, **1200**.

Wrong: 12.39765±0.063132 cm **Correct: (12.40±0.07)cm**

Rounding rules

1. Begin with computed values: $x = 0.1239765\,\mathrm{m}$, $\Delta x = 0.063132\,\mathrm{cm}$.
2. If the first significant figure is 1, ceil Δx to two significant digits. Otherwise ceil to one significant digit: $\Delta x = 0.07\,\mathrm{cm}$.
3. Set the common unit (and the exponent in scientific notation) to be the same in both x and Δx: $x = 12.39765\,\mathrm{cm}$.
4. Round normally x to the same number of decimal digits as the uncertainty: $x = 12.40\,\mathrm{cm}$.
5. Write the result as $x = (12.40 \pm 0.07)\,\mathrm{cm}$. Sometimes short form is possible: $x = 12.40(7)\,\mathrm{cm}$.

Intermediate calculations should be performed with at least one significant digit more.

Scientific notation

$$\text{mantisa} \times 10^{\text{exponent}}$$

$$1.2345 \times 10^{3}$$

- Convenient method to write very large or very small numbers.
- Exponent **must** be the same for the quantity and its uncertainty and written only once outside of the bracket.
- Mantissa should be something between 1 and 10 (in uncertainty between 0.1 and 10).
- Some conventions suggest exponent to be a multiply of 3.
- Sometimes it is better to use different units (e. g. µm instead of mm).
- Notation must be clear and legible

Some examples

Wrong	Correct
$U = (12.72434 \pm 0.62531)\,\mathrm{V}$	$\boldsymbol{U = (12.7 \pm 0.7)\,\mathrm{V}}$
$Q = \left(1.2{\times}10^{-3} \pm 5{\times}10^{-4}\right)\ \mathrm{mC}$	$\boldsymbol{Q = (1.2 \pm 0.5)\times 10^{-3}\,\mathrm{mC}}$
$I = 12.56\,\mathrm{mA} \pm 31\,\mu\mathrm{A}$	$\boldsymbol{I = (12.56 \pm 0.04)\,\mathrm{mA}}$
$j = (0.000001234 \pm 0.000000005)\,\frac{\mathrm{A}}{\mathrm{cm}^2}$	$\boldsymbol{j = (1.234 \pm 0.005)\times 10^{-6}\,\frac{\mathrm{A}}{\mathrm{cm}^2}}$
$p = (1129 \pm 80)\,\mathrm{hPa}$	$\boldsymbol{p = (113 \pm 8)\,\mathrm{kPa}}$

2 Experimental Errors and Uncertainties

2.1 Sources and Types of Errors

Sources of errors

- Subjective

  Results from the skill and attitude of the person running the experiment, ergonomy of the set-up, visibility etc. Usually their main source is lack of experience and bad habits:
  - premature rounding of the numbers,
  - autosuggestion (trying of confirming some supposition of the first measure),
  - on-the-fly calculations before noting down the result,
  - lack of patience (e. g. when heating liquids).
- Objective
  - quality and precision of the equipment,
  - type of the analysed phenomenon.

Types of errors

- Large errors / mistakes
- Systematic errors
- Random errors

Mistakes

Result from:

- wrong reading,
- mathematical mistake,
- carelessness,
- malfunction of the apparatus.

Can be detected during analysis. The wrong result clearly does not match the others.

Mistaken point must be immediately rejected. When necessary the measurements must be repeated.

Systematic errors

- Systematic errors give a constant shift to obtained results.
- They result from construction of the apparatus, chosen experimental method, or yet unknown or not considered physical phenomena.
- Hard to detect and eliminate.

Definition 3. Calibration refers to the process of determining the relation between the output (or response) of a measuring instrument and the value of the input quantity or attribute, a measurement standard.

Detecting and handling systematic errors

- Adjustment of the apparatus / calibration.
  - Should be performed (at least roughly) quite often and always when using newly constructed set-up.
- Additional methods:
  - verification of symmetry,
  - change of step order (hysteresis),
  - verification of stability of other parameters,
  - measurements of constant value to detect a time drift,
  - theoretical analysis or experimental verification (using reference values),
  - measurement of relative quantities.
- Systematic errors must be either eliminated or measured and included in the analysis of the results.

Random errors

- Unavoidable
- Subject of statistical analysis
- The value of the error must always accompany measured quantity
- Their existence implies that each measurement must be repeated several times

2.2 Error of a single measurement

Absolute and relative error

Each measurement has an error resulting from the precision of the instruments (e. g. ruler: $\Delta x = 1\,\mathrm{mm}$, calliper: $\Delta x = 0.02\,\mathrm{mm}$).

Definitions 4. The **absolute error** is the magnitude of the difference between the exact value and the approximation. $x \pm \Delta x$—the exact value somewhere in the range $[x - \Delta x, x + \Delta x]$.

The **relative error** is the ratio of the absolute error and the average value $\varepsilon = \Delta x / x$.

The real value of the measured quantity cannot be determined with total error smaller than the error of a single measurement (absolute error).

Propagation of absolute error

Assume that having some measured parameters x_1, x_2 etc. you want to find

$$y = f(x_1, x_2, \dots, x_n),$$

where every x_i has an absolute error Δx_i.

The absolute error of y is equal to

$$\Delta y = \left|\frac{\partial f}{\partial x_1}\right| \Delta x_1 + \left|\frac{\partial f}{\partial x_2}\right| \Delta x_2 + \ldots + \left|\frac{\partial f}{\partial x_n}\right| \Delta x_n.$$

The derivatives are computed at estimates of each x_i.

Propagation of absolute error: example

In tuned RLC circuit the Q-factor is

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

Assume that the following values are measured: $R = (50 \pm 2)\,\Omega$, $L = (200 \pm 5)\,\mathrm{mH}$, $C = (5.0 \pm 0.1)\,\mu\mathrm{F}$.

The absolute error of the Q-factor is

$$\Delta Q = \frac{1}{R^2}\sqrt{\frac{L}{C}}\,\Delta R + \frac{1}{2RC}\sqrt{\frac{C}{L}}\,\Delta L + \frac{L}{2RC^2}\sqrt{\frac{C}{L}}\,\Delta C$$

Propagation of absolute error: example

The approximated value (mind that $1\,\mathrm{H}/1\,\mathrm{F} = 1\,\Omega^2$)

$$Q = \frac{1}{50\,\Omega}\sqrt{\frac{200\,\mathrm{mH}}{5\,\mu\mathrm{F}}} = \frac{1}{50\,\Omega}\sqrt{\frac{200\times 10^{-3}\,\mathrm{H}}{5\times 10^{-6}\,\mathrm{F}}} = 4$$

The error

$$\Delta Q = \frac{1}{(50\,\Omega)^2}\sqrt{\frac{200\,\mathrm{mH}}{5\,\mu\mathrm{F}}} \times 2\,\Omega + \frac{1}{2\times 50\,\Omega \times 5\,\mu\mathrm{F}}\sqrt{\frac{5\,\mu\mathrm{F}}{200\,\mathrm{mH}}} \times 5\,\mathrm{mH}$$
$$+ \frac{1}{2\times 50\,\Omega\times (5\,\mu\mathrm{F})^2}\sqrt{\frac{5\,\mu\mathrm{F}}{200\,\mathrm{mH}}} \times 0.1\,\mu\mathrm{F} = 0.25$$

Hence

$$Q = 4.0 \pm 0.3$$

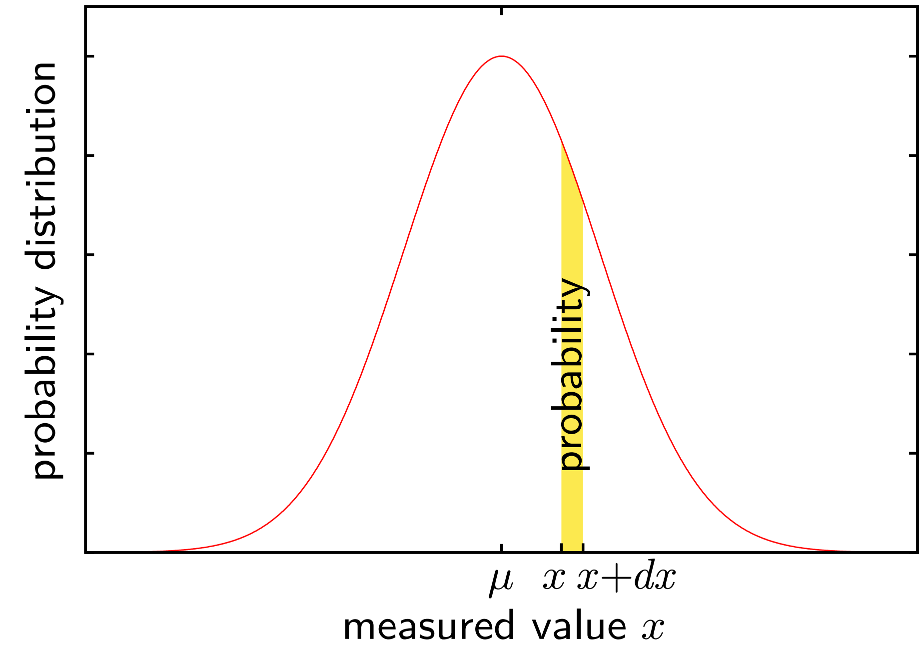
Propagation of error: basic operations

x, y, z are measured values with errors Δx, Δy and Δz, respectively. c and n are exact values.

function	**error**	
$f = x \pm y$	$\Delta f = \Delta x + \Delta y$	
$f = c\,x$	$\Delta f = \lvert c\rvert\,\Delta x$	$\frac{\Delta f}{f} = \frac{\Delta x}{x}$
$f = x\,y$	$\Delta f = \lvert x\rvert\,\Delta y + \lvert y\rvert\,\Delta x$	$\frac{\Delta f}{f} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$
$f = 1/x$	$\Delta f = \Delta x/x^2$	$\frac{\Delta f}{f} = \frac{\Delta x}{x}$
$f = x/y$	$\Delta f = \frac{\lvert x\rvert\,\Delta y + \lvert y\rvert\,\Delta x}{y^2}$	$\frac{\Delta f}{f} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$
$f = \sqrt{x}$	$\Delta f = \Delta x/(2\sqrt{x})$	$\frac{\Delta f}{f} = \frac{1}{2}\frac{\Delta x}{x}$
$f = x^n$	$\Delta f = \lvert n x^{n-1}\rvert\,\Delta x$	$\frac{\Delta f}{f} = n\frac{\Delta x}{x}$
$f = \mathrm{e}^{c\,x}$	$\Delta f = c\,\mathrm{e}^{cx}\Delta x,$	$\frac{\Delta f}{f} = c\Delta x$
$f = \log(c\,x)$	$\Delta f = \Delta x/x$	

2.3 Error of multiple measurements

Normal (Gaussian) distribution



Expected value

- If all of measures are equally significant than the best estimate of the expected value is **arithmetic average**

$$\mu \approx \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

- Statistical analysis can be used to estimate the accuracy of the measurements.

Definition 5. The **expected value** μ is the value arithmetic mean after infinite number of measurements. In normal (Gaussian) distribution it is the most probable value to obtain. It can be safely assumed that this is the “true” value of the quantity.

Random error and residual

Definition 6. A **statistical error** is the amount by which an observation differs from its expected value; the latter being based on the whole population from which the statistical unit was chosen randomly.

Definition 7. A **residual** (fitting error) is an observable estimate of the unobservable statistical error.

Variance and standard deviation of a single measurement

The **variance** gives is information about an expected error (*statistical dispersion*) of a single measure. It is defined as

$$\sigma_x^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

For practical reasons more useful is the **standard deviation**, which gives us information about the spread of each result in the same unit as the mean:

$$\sigma_x = \sqrt{\sigma_x^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Variance and standard deviation of a single measurement

Standard deviation:

$$\sigma_x = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

The above equation is true only when every single member of the population is studied! In practise the better estimate is the **sample standard deviation**:

$$s_x = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

From now on when talking about *standard deviation* we mean *sample standard deviation.*

Variance and standard deviation of the mean

In most cases we are not interested in the error of a single measurement but of the one of the mean. This information is given by the **standard error of the mean** defined as

$$S_x = \frac{s_x}{\sqrt{n}}$$

or

$$S_x = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Similarly we can define the **variance** of the mean:

$$S_x^2 = \frac{1}{n(n-1)}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

The practical consequence: **increasing the number of measurements we increase the precision of the result**.

Standard deviation: example

Consider mathematical pendulum o length $l = 1\,\mathrm{m}$. Measured values of the oscillation period are:

T [s]	$(T - \overline{T})^2$ [s^2]	**analysis**
2.01	0.0007	mean: $\overline{T} = 1.983\,\mathrm{s}$
1.94	0.0018	
2.09	0.0114	population variance: $\sigma_T^2 = 0.02668\,\mathrm{s}^2$
2.16	0.0313	population standard deviation: $\sigma_T = 0.164\,\mathrm{s}$
1.85	0.0177	
1.86	0.0151	sample variance: $s_T^2 = 0.02965\,\mathrm{s}^2$
2.14	0.0246	sample standard deviation: $s_T = 0.173\,\mathrm{s}$
2.24	0.0660	
1.83	0.0234	mean variance: $S_T^2 = 0.002965\,\mathrm{s}^2$
1.71	0.0745	mean standard deviation: $S_T = 0.055\,\mathrm{s}$

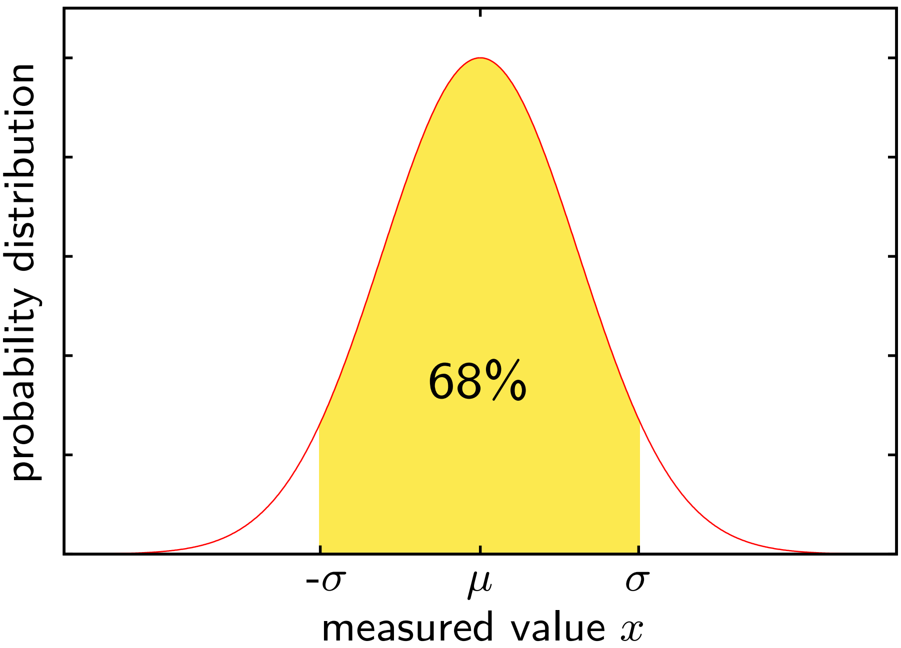
Interpreting standard deviation

So we know $\bar{x}$ and S_x. So what about the expected value μ?

Assume we know population standard deviation σ_x (impossible). The variable

$$z = \frac{\bar{x} - \mu}{\sigma_x/\sqrt{n}}$$

has normal distribution, so given some **confidence interval** we know how far the $\bar{x}$ is from μ.

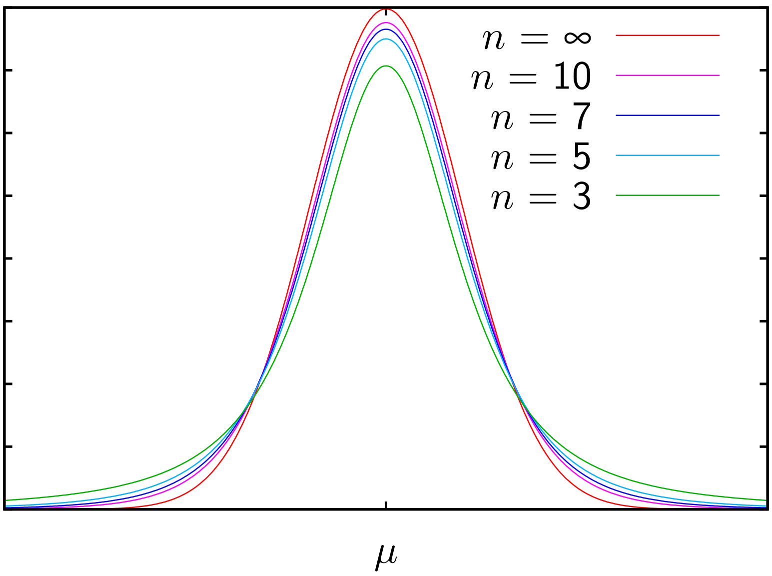


Student's t-distribution

But in reality we don't know σ_x. But we know s_x and S_x. Hence we can determine the variable

$$t = \frac{\bar{x} - \mu}{s_x/\sqrt{n}} = \frac{\bar{x} - \mu}{S_x}$$

It has distribution, which is close to the normal distribution... But not exactly. It has Student's t-distribution.



Interpretation

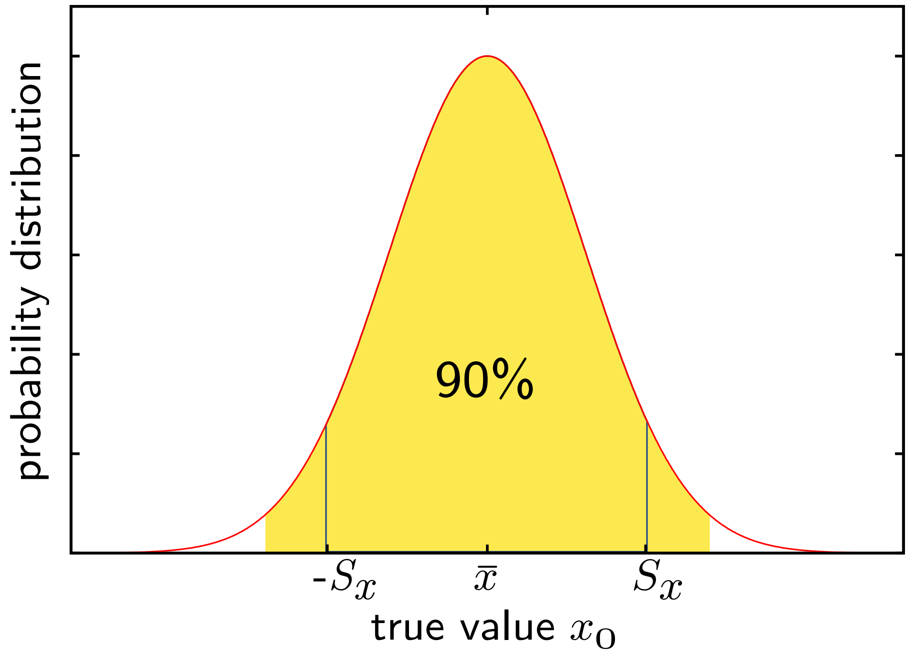
For large numbers of measurements n Student's distribution $\equiv$ normal distribution.

The measured average $\bar{x}$ belongs to the range $[\mu - S_x, \mu + S_x]$ with probability 68%. Or...
The true value $x_0 = \mu$ belongs to the range $[\bar{x} - S_x, \bar{x} + S_x]$ with probability 68%.

For small n this probability is even smaller (Student's distribution if "flatter").

If we want different (larger) probability, we have to multiply S_x by some factor $t_\alpha^{(n)}$. It depends on the number of degrees of freedom $n - 1$ and the desired probability (confidence interval) α.

Confidence intervals



Student's t-distribution coefficients ($t_\alpha^{(n)}$)

n	ν	50%	80%	90%	95%	99%	99.9%
2	1	1.000	3.078	6.314	12.71	63.66	636.6
3	2	0.820	1.886	2.920	4.303	9.925	31.60
4	3	0.765	1.638	2.353	3.182	5.841	12.92
5	4	0.741	1.533	2.132	2.776	4.604	8.610
6	5	0.727	1.476	2.015	2.571	4.032	6.869
7	6	0.718	1.440	1.943	2.447	3.707	5.959
8	7	0.711	1.415	1.895	2.365	3.499	5.408
9	8	0.706	1.397	1.860	2.306	3.355	5.041
10	9	0.703	1.383	1.833	2.262	3.250	4.781
∞	∞	0.674	1.282	1.640	1.960	2.576	3.291

For n measurements $x_0 = \bar{x} \pm t_\alpha^{(n)} S_x$ with probability α.

Number of *degrees of freedom* is $\nu = n - 1$

Rejecting wrong measurements

- Sometimes some measurements are far away from others.
- Should they be considered or rejected as mistakes?
- The simplest criterion:

$$\text{Reject } x \text{ if } |x - \bar{x}| > 3s_x.$$

- The measure not matching the others is called an **outlier**.

Error propagation of multiple measurements

Assume that you have some parameters $\bar{x}_1$, $\bar{x}_2$ etc., which are means of multiple measurements. Their standard deviations of the means (multiplied by $t_\alpha^{(n)}$ when needed) are S_1, S_2 etc.

You want to find

$$y = f(x_1, x_2, \ldots),$$

The statistical error of y is equal to

$$\Delta y = \sqrt{\left(\frac{\partial f}{\partial x_1} S_1\right)^2 + \left(\frac{\partial f}{\partial x_2} S_1\right)^2 + \dots}$$

and the derivatives are computed at $\bar{x}_1$, $\bar{x}_2$, ...

Propagation of statistical error: example

Again we compute the Q-factor or the RLC circuit:

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

The error of the Q-factor is

$$\Delta Q = \sqrt{\frac{L}{R^4 C} S_R^2 + \frac{1}{4R^2 LC} S_L^2 + \frac{L}{4R^2 C^3} S_C^2}.$$

Propagation of statistical error: basic operations

function	error	
$f = x \pm y$	$\Delta f^2 = \Delta x^2 + \Delta y^2$	
$f = c\,x$	$\Delta f^2 = c^2\,\Delta x^2$	$\left(\frac{\Delta f}{f}\right)^2 = \left(\frac{\Delta x}{x}\right)^2$
$f = x\,y$	$\Delta f^2 = x^2\,\Delta y^2 + y^2\,\Delta x^2$	$\left(\frac{\Delta f}{f}\right)^2 = \left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2$
$f = 1/x$	$\Delta f^2 = \Delta x^2/x^4$	$\left(\frac{\Delta f}{f}\right)^2 = \left(\frac{\Delta x}{x}\right)^2$
$f = x/y$	$\Delta f^2 = \frac{x^2\,\Delta y^2 + y^2\,\Delta x^2}{y^4}$	$\left(\frac{\Delta f}{f}\right)^2 = \left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2$
$f = \sqrt{x}$	$\Delta f^2 = \Delta x^2/4x$	$\left(\frac{\Delta f}{f}\right)^2 = \frac{1}{4}\left(\frac{\Delta x}{x}\right)^2$
$f = x^n$	$\Delta f^2 = \left\lvert n x^{n-1}\right\rvert^2\,\Delta x^2$	$\left(\frac{\Delta f}{f}\right)^2 = n^2\left(\frac{\Delta x}{x}\right)^2$
$f = \mathrm{e}^{c\,x}$	$\Delta f^2 = c^2\,\mathrm{e}^{2cx}\Delta x^2,$	$\left(\frac{\Delta f}{f}\right)^2 = c^2 \Delta x^2$
$f = \log(c\,x)$	$\Delta f^2 = \Delta x^2/x^2$	

Complete example

In the experiment the gravitational acceleration is measured with a mathematical pendulum. Both the length and the oscillation period are measured 10 times. Find $g = 4\pi^2 l/T^2$ with confidence level 95%.

l [m]	T [s]	**analysis**		
1.019	2.01			
0.996	1.94	mean:	$\bar{l} = 0.991\,\mathrm{m}$	$\overline{T} = 1.983\,\mathrm{s}$
1.011	2.09	mean standard deviation:	$S_l = 0.007\,\mathrm{m}$	$S_T = 0.055\,\mathrm{s}$
1.003	2.16			
1.000	1.85	for $P = 95\%$ and $n = 10$ Student's coefficient $\alpha_t = 2.262$		
0.998	1.86			
0.964	2.14	statistical error $\alpha_t\,S_x$:	$\Delta l = 0.014\,\mathrm{s}$	$\Delta T = 0.13\,\mathrm{s}$
0.967	2.24			
0.987	1.83	$l = (0.991 \pm 0.014)\,\mathrm{m}$		
0.968	1.71	$T = (01.98 \pm 0.13)\,\mathrm{s}$		

Complete example (cont.)

So by now we know that $T = (1.98 \pm 0.13)\,\mathrm{s}$ $l = (0.991 \pm 0.014)\,\mathrm{m}$

The gravitational acceleration is $g = 4\pi^2 l/T^2$. So

$$g = 4\pi^2 \frac{0.991\,\mathrm{m}}{(0.98\,\mathrm{s})^2} = 9.949\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

The error can be computed as

$$\Delta g = g\sqrt{\left(\frac{\Delta l}{l}\right)^2 + 4\left(\frac{\Delta T}{T}\right)^2}$$

$$\Delta g = 9.949\,\frac{\mathrm{m}}{\mathrm{s}^2}\sqrt{\left(\frac{0.014\,\mathrm{m}}{0.991\,\mathrm{m}}\right)^2 + 4\left(\frac{0.13\,\mathrm{s}}{1.98\,\mathrm{s}}\right)^2} = 1.32\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

Hence the result is: $\boldsymbol{g = (10.0 \pm 1.4)\,\frac{m}{s^2}}$.

3 Changing Parameters and Function Fitting

3.1 Correlation

Function fitting

- A typical aim of experiments is to find a relation between some quantities

$$y = f(x).$$

- Both x and y must be varied in order to determine the relation.
- Other parameters should remain constant (although it is good to repeat experiment for various values of other parameters).
- The relation can be predicted from the theory (desirable) or totally unknown.
- Determination of some constants in the experiment.

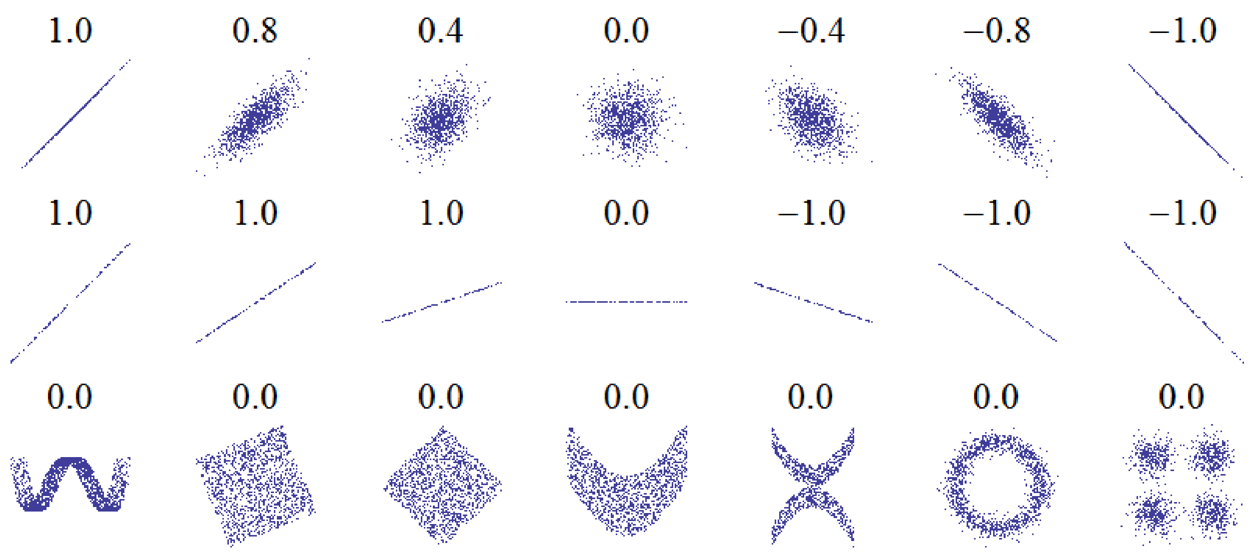
Fitting phases

1. Is there any relation between measured data? What kind of relation?
2. Choice of numerical method and determination of the parameters (with uncertainties of course).
3. Verification of the quality of fitting.

Correlation

Definition 8. **Correlation** (often measured as a **correlation coefficient**) indicates the strength and direction of a linear relationship between two random variables.

Correlation examples



Correlation coefficient

The best estimate of the correlation coefficient is **Pearson product-moment correlation coefficient**:

$$r_{xy} = \frac{\sum\limits_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{(n-1)\, s_x\, s_y} = \frac{\sum\limits_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \overline{x})^2}\,\sqrt{\sum_{i=1}^{n}(y_i - \overline{y})^2}}$$

- If $r \approx +1$ then we have positive correlation.
- If $r \approx 0$ then we have no correlation.
- If $r \approx -1$ then we have negative correlation.

3.2 Least Squares Method

Regression

Consider a set of pairs of experimental points $\{x_i, y_i\}$, for which we know (or assume) theoretical relation

$$y = f(x, a_0, a_{1,} a_{2,}, \ldots).$$

Our aim is to determine parameters a_0, a_1 etc. so that the function f matches the experimental points as well as possible, i. e. the differences between $f(x_i)$ and y_i are as small as possible.

This procedure is called **regression**. The most common (but not the only one) approach is the Least Squares Method.

Least Squares Method

Define a sum of squared differences

$$\chi^2 = \sum_{i=1}^{n} \left[y_i - f(x_i, a_0, a_1, \ldots)\right]^2 .$$

The aim is to find such parameters that S takes its minimal value. This is equivalent to searching the extremum by solving a set of equations:

$$\begin{aligned}
\frac{\partial \chi^2}{\partial a_0} &= 0 \\
\frac{\partial \chi^2}{\partial a_1} &= 0 \\
\frac{\partial \chi^2}{\partial a_2} &= 0 \\
&\vdots
\end{aligned}$$

Least Squares Method (cont.)

Parameters a_0, a_1 etc. can be determined (using analytical or numerical methods) as a function of x_i and y_i. In classical LSM it is assumed that the error of x_i is negligible. Then the error of the parameters is

$$\sigma_{a_k} = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial a_k}{\partial y_i}\right)^2 \sigma_y^2},$$

where σ_y is an expected value of the error of y.

Least Squares Method gives <u>wrong</u> results in case of the presence of outliers. Hence, the outliers must be identified and eliminated. The simplest way to do this is to make a plot.

Linearisation of a nonlinear function

Sometimes it is possible to transform a nonlinear function into a linear one. This allows to use linear least squares method.

Example 9. Consider function measurement of a viscosity as a function of temperature. The theoretical relation is

$$\eta = Ae^{\frac{W}{kT}}.$$

We are interested in the activation energy W. By taking logarithms of both sides of the above equation we have

$$\log \eta = \log A + \frac{W}{k} T^{-1}.$$

Taking $x \to T^{-1}$, $y \to \log \eta$, $a \to W/k$ and $b \to \log A$ we have linear equation

$$y = ax + b \quad \text{or better} \quad \log \eta = a\,T^{-1} + b.$$

Linearisation of typical functions

function	linear function $\psi = a\xi + b$	substitutions			
		ψ	ξ	a	b
$y = A\,x^n + B$	$y = A\,x^n + B$	y	x^n	A	B
$y = A\,B^{Cx}$	$\log y = C \log B\,x + \log A$	$\log y$	x	$C \log B$	$\log A$
$y = A\,e^{Cx}$	$\log y = C\,x + \log A$	$\log y$	x	C	$\log A$
$y = A\,x^B$	$\log y = B\,\log x + \log A$	$\log y$	$\log x$	B	$\log A$

Linear regression

Consider the set of n pairs $[x_i, y_i]$ expected to fulfil

$$y = ax + b.$$

Our task is to find a and b together with their errors.

We have

$$\chi^2 = \sum_{i=1}^{n} (y_i - ax_i - b)^2 = \sum_{i=1}^{n} \left(y_i^2 + a^2 x_i^2 + b^2 - 2ax_i y_i - 2by_i + 2abx_i \right).$$

Linear regression (cont.)

Define: $S_x = \sum_{i=1}^n x_i$, $S_y = \sum_{i=1}^n y_i$, $S_{xx} = \sum_{i=1}^n x_i^2$, $S_{xy} = \sum_{i=1}^n x_i y_i$. Now we can write:

$$\chi^2 = S_{yy} + a^2 S_{xx} + nb^2 - 2aS_{xx} - 2bS_y + 2abS_x.$$

The derivatives are:

$$\begin{aligned}
\frac{\partial \chi^2}{\partial a} &= 2aS_{xx} - 2S_{xy} + 2bS_x = 0 \\
\frac{\partial \chi^2}{\partial b} &= 2nb - 2S_y + 2aS_x = 0
\end{aligned}$$

Solving these two equations we have:

$$\begin{aligned}
a &= \frac{nS_{xy} - S_x S_y}{\Delta}, \\
b &= \frac{S_{xx} S_y - S_x S_{xy}}{\Delta},
\end{aligned}$$

where $\Delta = nS_{xx} - S_x S_x$.

Linear regression: error

$a = (nS_{xy} - S_xS_y)/\Delta$, $b = (S_{xx}S_y - S_xS_{xy})/\Delta$, $\Delta = nS_{xx} - S_xS_x$.

The error can be computed assuming that the expected value of σ_y is

$$\sigma_y = \sqrt{\frac{\chi^2}{n-2}}.$$

So we have:

$$\begin{aligned}
\sigma_a &= \sigma_y \sqrt{\sum_{i=1}^{n}\left(\frac{\partial a}{\partial y_i}\right)^2} = \sigma_y \sqrt{\frac{\sum_{i=1}^{n}\left(n\frac{\partial}{\partial y_i}S_{xy} - \frac{\partial}{\partial y_i}S_xS_y\right)^2}{\Delta^2}},\\
\sigma_b &= \sigma_y \sqrt{\sum_{i=1}^{n}\left(\frac{\partial b}{\partial y_i}\right)^2} = \sigma_y \sqrt{\frac{\sum_{i=1}^{n}\left(\frac{\partial}{\partial y_i}S_{xx}S_y - \frac{\partial}{\partial y_i}S_xS_{xy}\right)^2}{\Delta^2}}.
\end{aligned}$$

Linear regression: error (cont.)

Let's compute these derivatives:

$$\begin{aligned}
&\frac{\partial}{\partial y_i}S_{xy} = \sum\nolimits_{j=1}^{n}\frac{\partial}{\partial y_i}x_jy_j = x_i,\\
&\frac{\partial}{\partial y_i}S_xS_y = S_x\frac{\partial}{\partial y_i}S_y = S_x\left(\sum\nolimits_{j=1}^{n}\frac{\partial}{\partial y_i}y_j\right) = S_x,\\
&\frac{\partial}{\partial y_i}S_{xx}S_y = S_{xx}\frac{\partial}{\partial y_i}S_y = S_{xx},\\
&\frac{\partial}{\partial y_i}S_xS_{xy} = S_x\frac{\partial}{\partial y_i}S_{xy} = S_xx_i.
\end{aligned}$$

Linear regression: error (cont.)

This gives us:

$$\begin{aligned}
\sigma_a &= \sigma_y\sqrt{\frac{\sum_{i=1}^{n}(nx_i - S_x)^2}{\Delta^2}} = \sigma_y\sqrt{\frac{\sum_{i=1}^{n}(n^2x_i^2 - 2nx_iS_x + S_xS_x)}{\Delta^2}}\\
&= \sigma_y\sqrt{\frac{n^2S_{xx} - 2nS_xS_x + nS_xS_x}{\Delta^2}} = \sigma_y\sqrt{\frac{n}{\Delta}}
\end{aligned}$$

and:

$$\begin{aligned}
\sigma_b &= \sigma_y\sqrt{\frac{\sum_{i=1}^{n}(S_{xx} - S_xx_i)^2}{\Delta^2}}\\
&= \sigma_y\sqrt{\frac{\sum_{i=1}^{n}(S_{xx}S_{xx} - 2S_{xx}S_xx_i + S_xS_xx_i^2)}{\Delta^2}}\\
&= \sigma_y\sqrt{\frac{nS_{xx}S_{xx} - 2S_{xx}S_xS_x + S_xS_xS_{xx}}{\Delta^2}} = \sigma_y\sqrt{\frac{S_{xx}}{\Delta}}
\end{aligned}$$

Confidence intervals of linear regression coefficients

Determined coefficients a and b based on the sample have Student's distribution with $n-2$ degrees of freedom (equivalent to $n-1$ measurements).

Hence for confidence interval α:

$$\begin{aligned}
a &= a \pm t_\alpha^{(n-1)}\sigma_a\\
b &= b \pm t_\alpha^{(n-1)}\sigma_b
\end{aligned}$$

Linear regression: summary

$$\begin{aligned} a &= \frac{nS_{xy} - S_x S_y}{\Delta} \pm t_\alpha^{(n-1)} \sigma_a, \\ b &= \frac{S_{xx} S_y - S_x S_{xy}}{\Delta} \pm t_\alpha^{(n-1)} \sigma_b, \\ \sigma_a &= \sigma_y \sqrt{\frac{n}{\Delta}}, \\ \sigma_b &= \sigma_y \sqrt{\frac{S_{xx}}{\Delta}}, \end{aligned}$$

where: $\Delta = nS_{xx} - S_x S_x$,

$\sigma_y = \sqrt{\frac{\chi^2}{n-2}} = \sqrt{\frac{1}{n-2} \sum_{i-1}^{n} (y_i - ax_i - b)^2}$,

$S_x = \sum_{i=1}^n x_i$, $S_y = \sum_{i=1}^n y_i$, $S_{xx} = \sum_{i=1}^n x_i^2$, $S_{xy} = \sum_{i=1}^n x_i y_i$.

Repeated measurements vs. LSM

- Very often experiments can be designed to either use multiple measurements of the same quantity or least squares method.
- In such case which method is more accurate?

Consider measurements of gravitational acceleration using simple pendulum. In the first case the measurements are repeated 7 times with a pendulum of length l and g is computed as

$$g = 4\pi^2 l / T^2.$$

In the second measurement we repeat the same experiment by varying l. Then we have

$$T^2 = \frac{4\pi^2}{g} l = a\, l$$

and $g = 4\pi^2/a$. Confidence intervals is 90% in both cases.

Repeated measurements vs. LSM: results
Multiple measurements (for $l = 1\,\mathrm{m}$)

T [s]	1.966	2.029	1.990	1.978	1.994	1.968	2.026

$\overline{T} = 1.993\,\mathrm{s}$, $S_T = 0.010\,\mathrm{s}$, $t_\alpha^{(7)} = 1.943$ so $T = (1.993 \pm 0.019)\,\mathrm{s}$.
Hence:

$$g = (9.9 \pm 0.2)\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

as $\Delta g/g = 2\,\Delta T/T$.

Least squares method

l [m]	0.500	0.600	0.700	0.800	0.900	1.000	1.100
T [s]	1.426	1.548	1.676	1.793	1.899	2.009	2.106

Using relation $T^2 = a\, l + b$ and least squares method we have:
$a = 4.030\,\frac{\mathrm{s}^2}{\mathrm{m}}$, $b = 0.005\,\mathrm{s}^2$, $\sigma_a = 0.031\,\frac{\mathrm{s}^2}{\mathrm{m}}$, $\sigma_b = 0.025\,\mathrm{s}^2$, $t_\alpha^{(7-1)} = 2.015$.
So $a = (4.03 \pm 0.07)\,\frac{\mathrm{s}^2}{\mathrm{m}}$ and

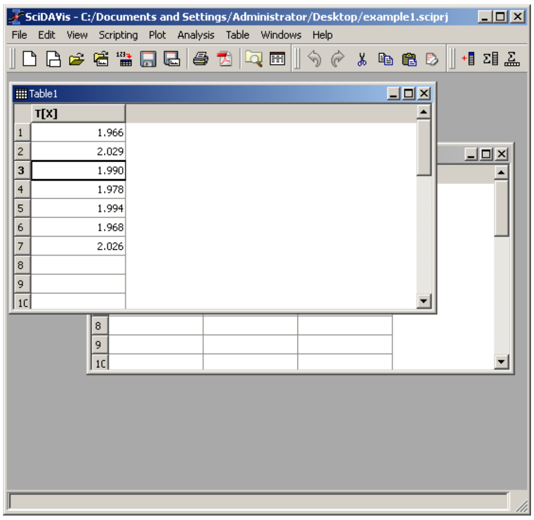
$$g = (9.80 + 0.15)\,\frac{\mathrm{m}}{\mathrm{s}^2}.$$

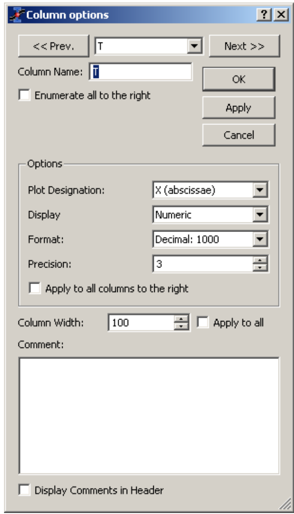
3.3 Statistical Analysis and LSM using SciDaVis

SciDAVis

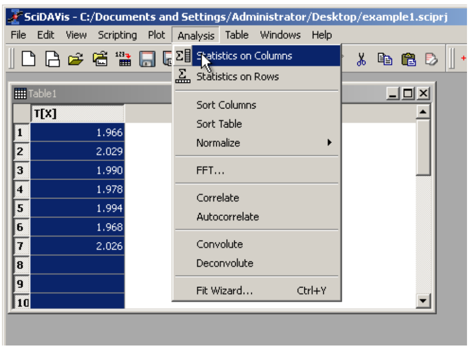
- **SciDAVis** is a free interactive application aimed at data analysis and publication-quality plotting.
- Shallow learning curve and an intuitive, easy-to-use graphical user interface.
- Powerful features such as scriptability and extensibility.
- Runs on GNU/Linux, Windows and MacOS X.
- Similar to proprietary Windows applications like **Origin** and **SigmaPlot** as well as free applications like **QtiPlot**, **Labplot** and **Gnuplot**.
- Project homepage: http://scidavis.sourceforge.net
- Binary download: http://phys.p.lodz.pl/info/mdems

SciDAVis: tables





SciDAVis: statistics



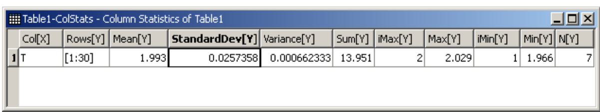
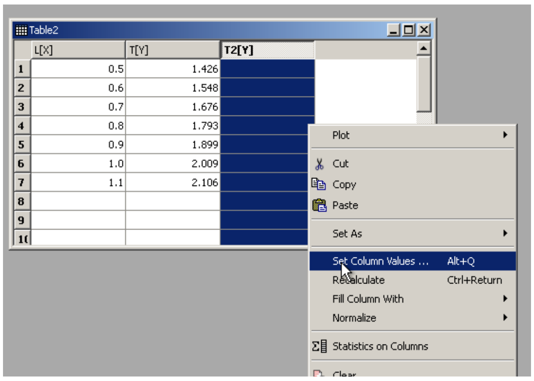
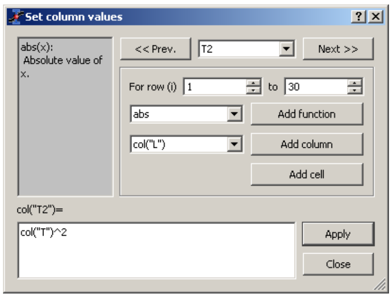


Table1-ColStats - Column Statistics of Table1

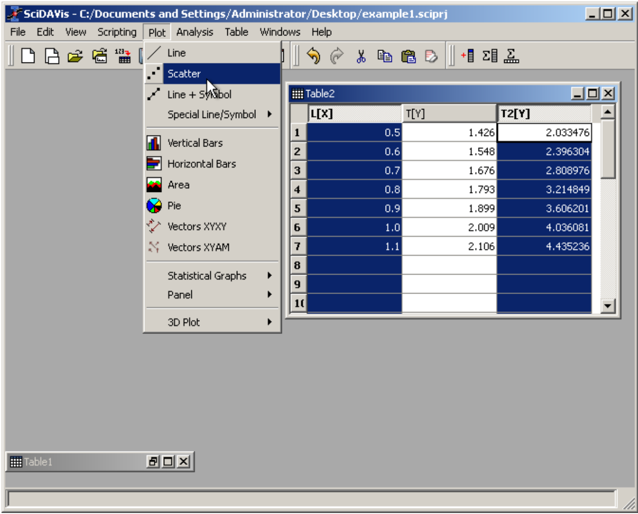
	Col[X]	Rows[Y]	Mean[Y]	StandardDev[Y]	Variance[Y]	Sum[Y]	iMax[Y]	Max[Y]	iMin[Y]	Min[Y]	N[Y]
1	T	[1:30]	1.993	0.0257358	0.000662333	13.951	2	2.029	1	1.966	7

SciDAVis: calculations on columns

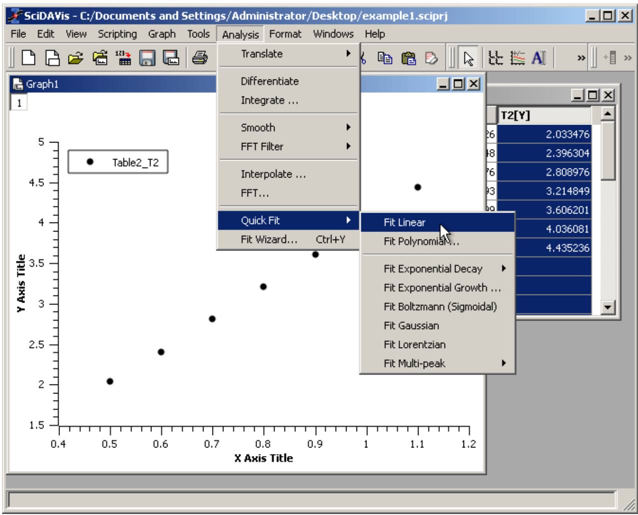




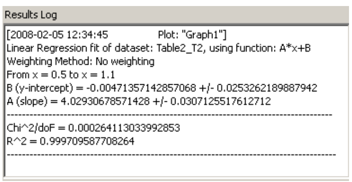
SciDAVis: making plot



SciDAVis: linear fit



SciDAVis: linear fit results



The presented results should be read as:

$$
\begin{aligned}
a &= 4.029 \qquad \sigma_a = 0.031 \\
b &= -0.005 \qquad \sigma_b = 0.026
\end{aligned}
$$

The correlation is big ($R^2 = 0.9997$). **Don't forget about units!**

Similarly you can do polynomial and may other fits. Much better than doing it by hand or by using Excel.

Part II
Presentation of Results

Presentation of Results: Outline

Contents

4 The Perfect Report

4.1 Report Structure

Experiment report

- The aim of the report is to present your results in a **clear and consise** way.
- You shoud address it to a reader with general knowledge in physics but not necessary familiar with your experiment.
- Give the necessary background, but shortly.
- You are to present your results, not the whole human knowledge in the subject.
- Laboratory report is neither a textbook, nor labnotes.
- Use good style:
  - each table and figure has its number and caption,
  - each table, figure and bibliographic item have to be referrenced in the main text,

Report layout

1. Headline
2. Abstract
3. Introduction
4. Theory
5. Short description of the experimental method
6. Experimental results and analysis
7. Conclusions
8. Bibliography
9. Appendinx (when needed)

Headline

Exercise 123

Measuring something and the other thing

Winnie the Pooh and Harry Potter

1st March 2008

Abstract

- Short description of the exercise done and the main results.
- No more than 5 sentences.

Abstract

In the experiment the doping profile of semiconductors was measured by taking the coltage-capacitance characteristics of a diode in the reversed bias. The measurements were performed for a photodiode and Schottky metal-silicon junction. It has been found that the method is precise only when the doping is constant. The results were compared with the doping density obtained from measuring the resistivity of the silicon.

Introduction

- Define the problem and state your goal.
- Show why do you do the experiment. Why this is relevant? What the results can be used for?
- Introduce the most important terminology.
- Show some background, history, present knowledge you want to expand etc.
- Provide a road-map and briefly state a structure of your document.
- Be consise. No more than a page.

Theory

- You should present all the theory, which is necessary to understand your work and which you refer to.
- All the equations you use should come here.
  - But do not exgearate. Don't give equations for the arithmetic mear or standard deviation. It is sufficient to name them.
  - Each equation should be numbered in braces at the right side: (1). Use this number for a reference in the text.
- Give only the necessary things. Make *citations*to more general informations:
  - Citation is a number of a position in your *bibliography*containing the necessary information.
  - You should put this number in square brackets: [1].
  - Every non-obvious information or equation rewritten from some other source must have proper citation.

  The relation between the boiling temperature and gas pressure is described by a Claussius-Clapeyron equation [2]:

Short description of the experimental method

- Present the key points of your experimental method: apparatus, main measurements, some <u>not obvious</u> tricks to improve accuracy.
- You should refer to the theory section. Show which equations you use in which stage.
- Make a schematic diagram of your set-up.
- Don't give details. Nobody is interested in the serial number of the oscilloscope.

Experimental results and analysis

- Here you present your results.
- You must give measured values, and their uncertainties.
- If possible put your results into tables.
  - Clearly mark which quantities has been measured and which calculated (how).
  - Keep the proper number of significant digits in the tables.
  - Choose such units that the measured valuer are in range arounf 1–100. Put the units in the table headers.
- After tables you show analysis: compute means, errors, substitute into equations, LSM etc.
  - Do not put all the detailed calculations here. Just show the starting point and give the result.

Conclusions

- The most important part of the report.
- Here you should indicate whether you have realized your goal or not (in which case show why).
- What you have learned? How the measurement can be done better.
- You should compare your results with the ones published in the literature if possible.

Bibliography

- Always put bibliogrphy in the end of the report!
- Each item has to be referenced in the main text.

[1] B. Żółtowski, M. Dems, *Experimental physics*, script for students of Technical University of Łódź, Łódź 2008.
[2] A. Taflove, *Computational Electrodynamics: The finite-difference time-domain method*—2nd edition, Artech House, Boston 2000.
[3] Lord Rayleigh, On the maintenance of vibrations by forces of double fre- quency, and on the propagation of waves through a medium endowed with a periodic structure, *Phil. Mag.*, vol. 27, pp. 145–159, 1887.

Appendix

- Put here everything you consided important but what would distract the reader in the main text.
  - some your own derivations
  - proofs of used theorem
- Put only your contributions. Do not rewrite textbooks, internet etc. (in such case citation is sufficient).

4.2 Typographic Conventions

General rules

- Printing now is much easier than it used to be.
- There are rules you need to obey.
- Keep your work nicely-looking and legible.
- Mind that Polish and English typographic rules are different!
- Use serif font in printed text. You may use sans-serif in headlines, presentations etc.
- Don't change the the font unneceserly. Every document can be written using only one font (e. g. Times).
- Avoid using colour and other "ornaments".

Dots, commas etc.

Full stops, commas, etc.: always put right after the previous word and put the space afterwards.

wrong	**correct**
‘Mrs.David Copperfield ,I think ,’ said Miss Betsey ; the emphasis referring ,perhaps,to my mother’s mourning weeds , and her condition .	‘Mrs. David Copperfield, I think,’ said Miss Betsey; the emphasis referring, perhaps, to my mother’s mourning weeds, and her condition.

Ellipsis: denote rhetorical pause or omission in quotation; don’t use three dots instead; don’t put spaces around.

wrong	**correct**
Dust...American...Dust	Dust. . . American. . . Dust

Quotations, parenthesis

You cannot put space inside quotation marks nor parenthesis. There must be a space outside of them.

Put the finishing dots, commas etc. inside quotations and outside parenthesis.

wrong	**correct**
‘ Well, well! ’ said Miss Betsey. ‘ Don’t cry any more ’.	‘Well, well!’ said Miss Betsey. ‘Don’t cry any more.’
This is shown in the diagram (see Fig. 1.)	This is shown in the diagram (see Fig. 1).
”something”	“something”
„something” (Polish version)	“something”

Dashes

wrong	**correct**
m-dash (—): separates parts of the sentence or denotes omissions	
Fiction-if it aspires to be art — appeals to temperament.	Fiction—if it aspires to be art—appeals to temperament.
Don’t blame me when the s- hits the fan.	Don’t blame me when the s— hits the fan.
n-dash (–): denotes intervals	
August 12-14	August 12–14
New York-Miami train	New York–Miami train
hyphen (-): appears in complex words	
black–or–white	black-or-white
minus (−): used in math	
$\mathrm{e}^{\mathrm{i}\pi}$ = -1, $\mathrm{e}^{\mathrm{i}\pi}$ = —1	$\mathrm{e}^{\mathrm{i}\pi} = -1$

Typesetting math

- Use italic for denoting scalar values.
- Use boldface font for denoting vectors.
- Don’t use sans-serif fonts in equations.
- Use roman font for denoting units. Don’t put units in brackets [ ] (instead of plots and table headers).
- Use roman fonts for function names sin, cos, etc.
- You may use roman characters for Euclid constant, imaginary unit and d in derivatives: $\mathrm{d}y/\mathrm{d}x = A\mathrm{e}^{\mathrm{i}\varphi}$. But be consistent!
- **Never** use asterisk (*) for denoting multiplication. Use $\times$, $\cdot$, or nothing instead. Asterisk means convolution or conjugate.
- Don’t use capital Π if you mean $\pi \approx 3.14$.
- Don’t change the size of the equations unnecessarily.

Typesetting math: examples

wrong	correct
italic symbols	
$\mathsf{I} - \mathsf{l}$	$I - l$
$\mathrm{d} = 1/\mathrm{l}$	$d = 1/l$
bold symbols for vectors	
$F = qv \times B$	$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$
units	
$I = 2\,mA$, $I = 2\ [mA]$, $I = 2\ [\mathrm{mA}]$	$I = 2\,\mathrm{mA}$
functions	
$e^{i\varphi} = cos\varphi + isin\varphi$	$\mathrm{e}^{\mathrm{i}\varphi} = \cos\varphi + \mathrm{i}\sin\varphi$ $e^{i\varphi} = \cos\varphi + i\sin\varphi$
asterisk	
$z^* * z = \|z\|^2$	$z^* z = \|z\|^2$

5 Presenting Numerical Data

5.1 Methods of presenting data

Presenting Numerical Data

- Tables
- Plots

Each table and plot should be located in an *inset*, i. e. separated element of the report with its own number (used for referrencing) and caption.

Plot inset

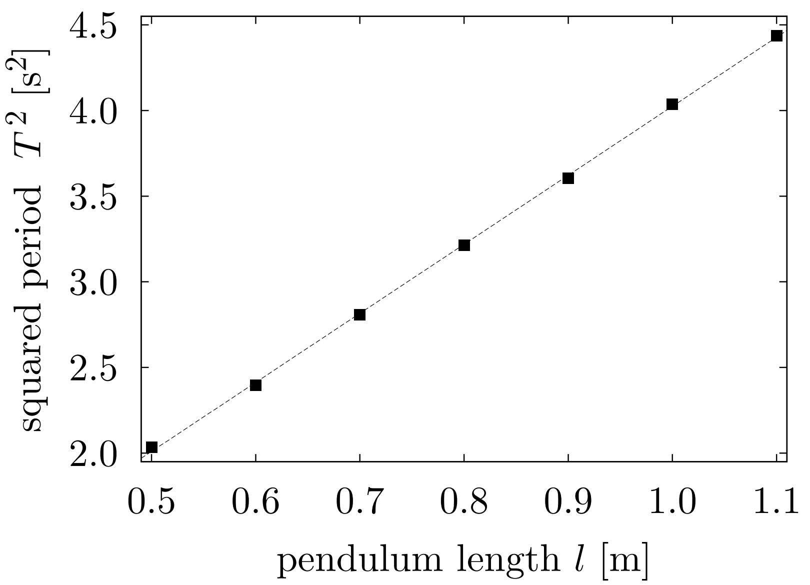


Figure 1: A measured relation between the squared pendulum oscillation period T^2 and thelength of the pendulum l. The line indicates function fitted with LSM.

Table inset

Table 1: Measured values of oscillation T periods for various lengths of the pendulum l. The values of T^2 are calculated form the measured quatities.

l [m]	T [s]	T^2 [s^2]
0.5	1.426	2.0335
0.6	1.548	2.3963
0.7	1.676	2.8090
0.8	1.793	3.2148
0.9	1.899	3.6062
1.0	2.009	4.0361
1.1	2.106	4.4352

5.2 Making Tables

Rules on making tables

- Put values of the sequential measurements of some value in table columns.
- Make good headers: quantity *symbol* [unit]. Use the name in the caption
- Choose unit of the power in scientific notation so that the value in table is in range 0.1–1000.
- Make the table easy to read. Don't make long tables of two columns only spanning multiple pages. Use the horizontal space you have.

h_i [kPa]	p_i [kPa]	T_i [°C]	T_i [K]	$\log p_i$	T_i^{-1} [10^{-3} K^{-1}]
71.4	29.0	71	344	3.368	2.907
62.0	38.4	77	350	3.648	2.857

5.3 Using Plots

Advantages of using plots

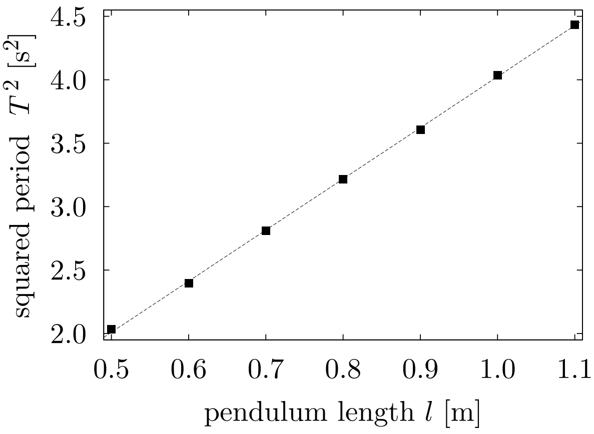
- Easy to understand way of presenting data.
- Can give information about the character of the relation between two quantities (linear, exponential, quadratic).
- Allows to visually detect the wrong measurements.

Making plots

- Choose proper plot ranges
- Make good axes
- Put experimental results in "scater plot"
- Add continous lines for theoretical curves.

Plot ranges

- Determine in what range your variables vary.
- Choose the plotting ranges to properly illustrate the chatacter of changes.
- You plot should connect one corner with the opposite one (more less).
- **Your plot does not need to begin in 0.**



Axes

- The independent (controled) value should be on the horizontal axis and the measured one on the vertical axis.
- The values on the axes should be in range 0.1–100. Choose proper units or powers in scientific notation.
- Make the tics at full 0.1, 0.2 0.5, 1, 2, 5, 10, 20, 50 etc.
- The density of tics must make the plot legible.
- Don't make tics at data points!
- Name the axes: give short variable name *symbol* and [unit] (and 10^{power} when needed).
- If the units are unknown use *arbitrary units* [a.u.]

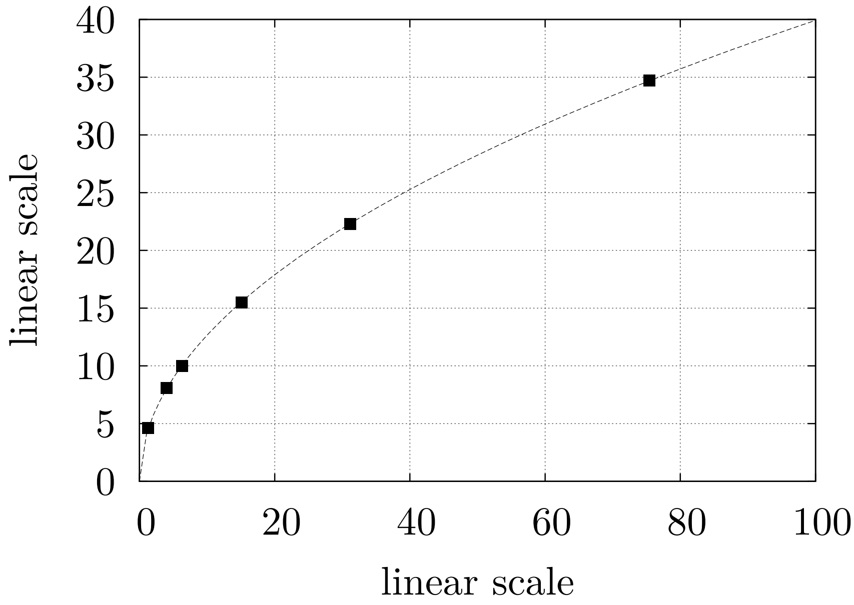
Putting experimental data

- Put the measured data using single points. **Never connect the points with straigth lines!**
- You may put the results of several measurements (e. g. repeated in different temperatures) but use different point symbols and add the legend. Single plot does not need a legend.
- Put theoretical relations or curves estimated with LSM using continous line.
- You may put information about uncertainties using small rectangles or crosses aroud each point.

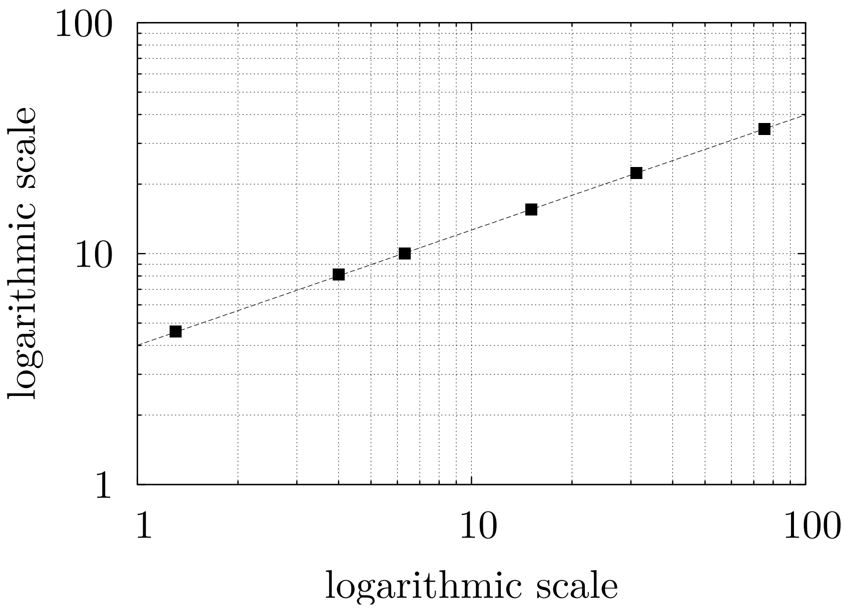
Types of plot scales

- Linear
- Logarithmic
  - good for exponential relations
  - useful for showing both very small and very large values
  - good for showing relative changes (increase by some factor is a shift in logarithmic scale)
- Polar
  - used for showing dependence on the angle

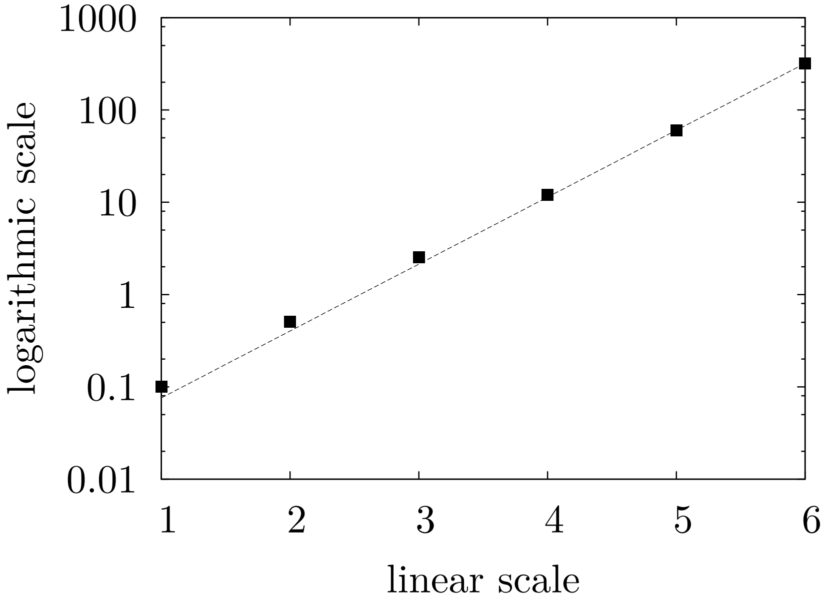
Linear scale



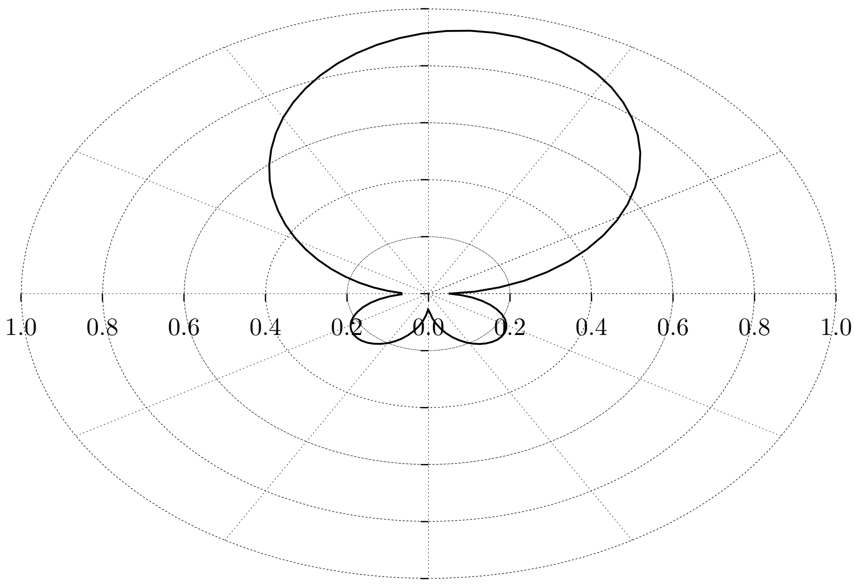
Logarithmic scale in x- and y-axis



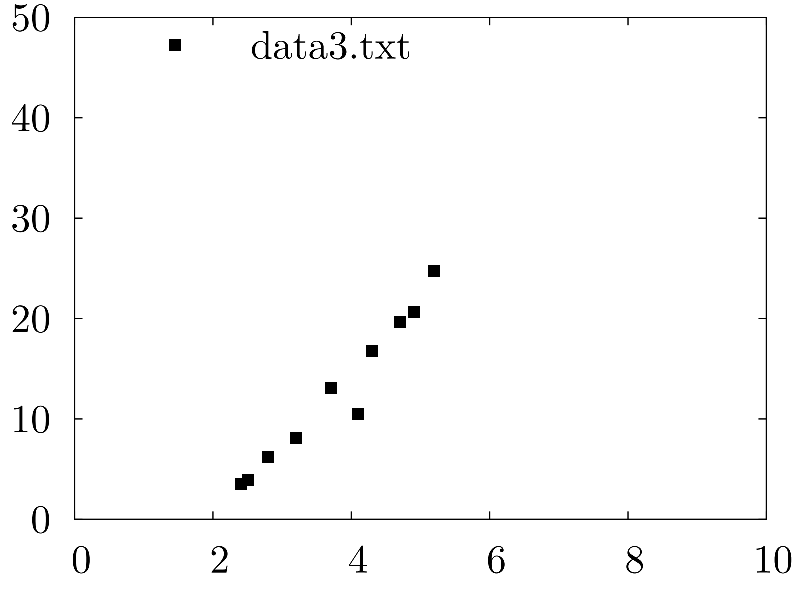
Logarithmic scale in y-axis only



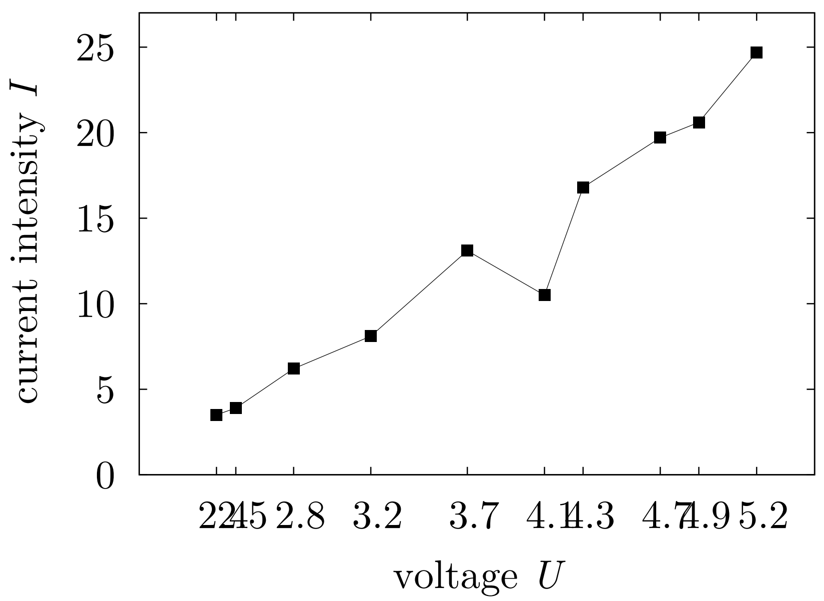
Polar scale



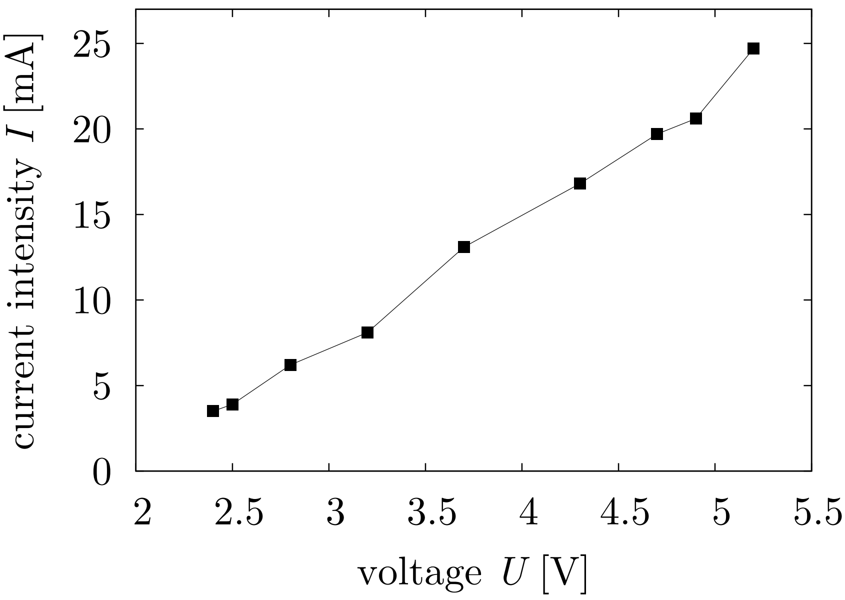
What's wrong with this?



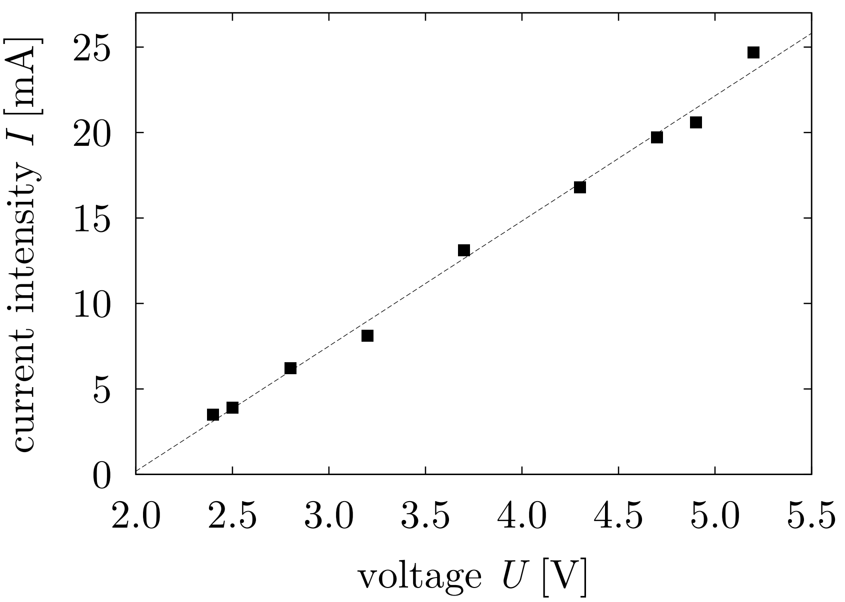
What's wrong with this?



What's wrong with this?



Correct plot should look like this



6 Useful Tools

6.1 LaTeX and LyX

What is LaTeX?

Definition 10. LaTeX is a document markup language and document preparation system for the TeX typesetting program. It is most widely used by mathematicians, scientists, philosophers, engineers, scholars in academia and the commercial world, and other professionals. LaTeX is used because of the quality of typesetting and extensive facilities for automating most aspects of typesetting and desktop publishing, including numbering and cross-referencing, tables and figures, page layout and bibliographies.

Advantages of LaTeX

- You think about WHAT to write and not HOW it should look. The layout is done for you.

- The output looks much proffesional than using Word.
- You can write your report in notepad (of course there are much better tools and most of them are free).
- Automatic TOC creation, equation and figure numbering etc.
- Simple and powerful typesetting of mathematical equations. No more MathType ordeal!

Although:

- You have to get used to not immidiately seeing the final result (similar to programming).
- You have to learn using it. . . but once you do you never go back to Word.

Simple document

```
\documentclass{article}
\usepackage[a4paper,margin=2cm]{geometry}
\title{A Very Simple \LaTeX{} Example}
\author{Maciej Dems}

\begin{document}
\maketitle

\section{Introduction}

This is time for all good men to come to the aid
of their party! And here your adventure begins.

To compute $\sqrt{\pi}$ use the following example
\begin{equation}\label{eq:gaussian}
  \int_{-\infty}^{+\infty} e^{-x^2} = \sqrt{\pi}
\end{equation}

\end{document}
```

The output

A Very Simple LaTeX Example

Maciej Dems

February 8, 2008

1 Introduction

This is time for all good men to come to the aid of their party! And here your adventure begins.

To compute $\sqrt{\pi}$ use the following example

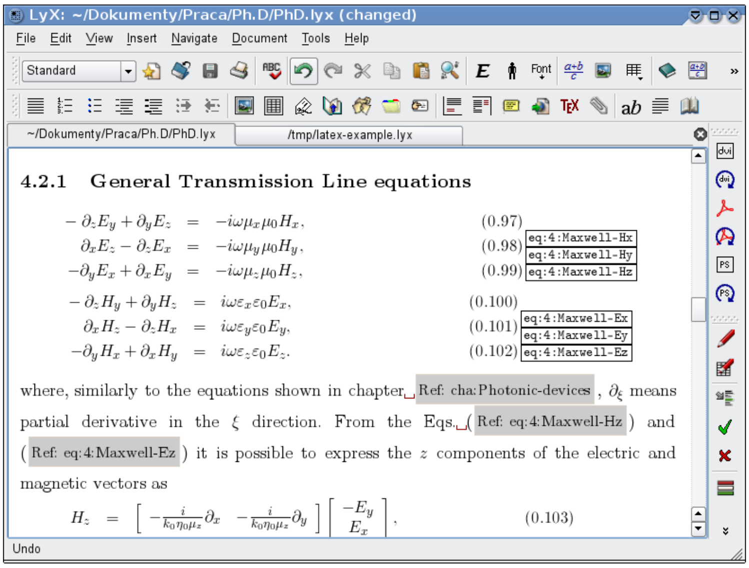
$$\int_{-\infty}^{+\infty} e^{-x^2} = \sqrt{\pi} \tag{1}$$

1

How about graphics?

- Graphics have to be saved in external files.
- Preffered format is encapsulated postsctipt (EPS).
- Every good vector-drawing program can export to EPS.
- Also good scientific plotting programs (like SciDAVis or gnuplot) can export to EPS.
- Figures (and tables) can be inserted directly or placed on insets with automatic numbering and labels for referencing.

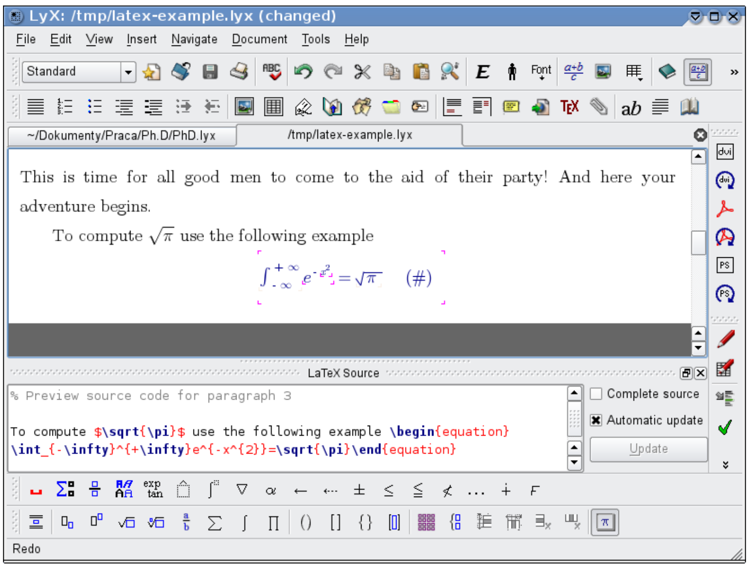
LyX: graphical interface to LaTeX



WYSIWYM

- **WYSIWYM**: What You See Is What You Mean.
- The document on screen resembles the printed one
- You define the **meaning** of the elements in your document: title, headers, listst, numbering, bibliography etc.
- LaTeX creates the final document for you.
- Much easier to learn and still offers the power of LaTeX.

LyX generates LaTeX file for you

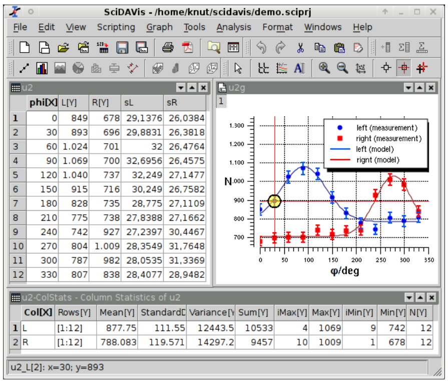


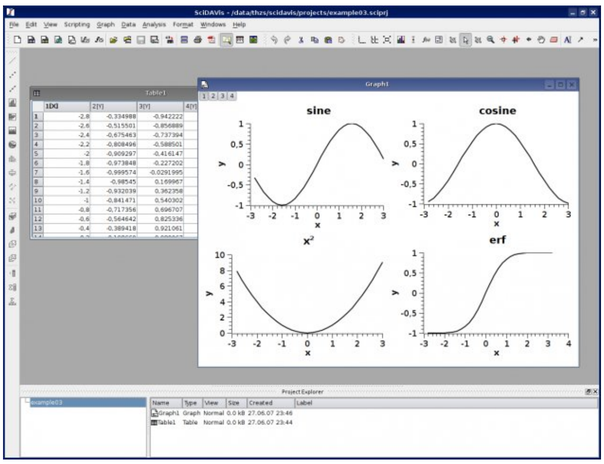
How to get the stuff?

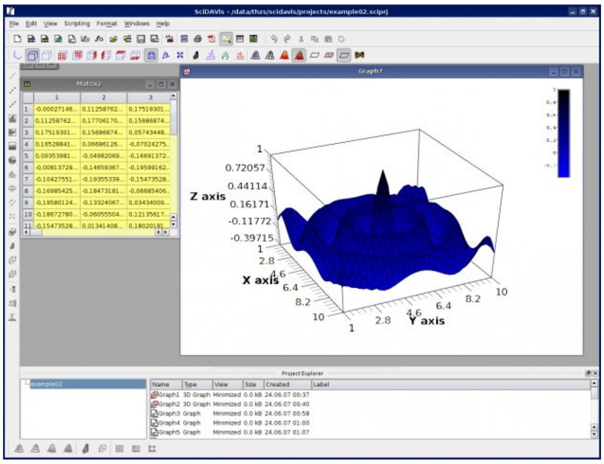
- LaTeX
  - Present in every major Linux distribution.
  - In Windows use MikTeX: http://miktex.org/
- LaTeX editors for Windows:
  - WinEdt (http://www.winedt.com)
  - TeXnicCenter (http://www.toolscenter.org)
- LyX
  - LyX homepage: http://www.lyx.org
  - Windows installer: http://wiki.lyx.org/Windows/Windows
- Good LaTeX course (in Polish):
  - Nie za krótkie wprowadzenie do systemu LaTeX 2_ε: http://www.ctan.org/get/info/lshort/polish/lshort2e.pdf http://tinyurl.com/2lodgv

6.2 SciDAVis

SciDAVis







6.3 Word? Excel?

LaTeX vs. Word

Introduction to LaTeX
©2006 by Harvey Gould
December 5, 2006

1 Introduction

TeX looks more difficult than it is. It is almost as easy as π. See how easy it is to make special symbols such as α, β, γ, δ, $\sin x$, $\hbar$, λ, ... We also can make subscripts A_x, A_{xy} and superscripts, e^x, e^{x^2}, and e^{a^b}. We will use LaTeX, which is based on TeX and has many higher-level commands (macros) for formatting, making tables, etc. More information can be found in Ref. [1].

We just made a new paragraph. Extra lines and spaces make no difference. Note that all formulas are enclosed by $ and occur in *math mode*.

The default font is Computer Modern. It includes *italics*, **boldface**, *slanted*, and `monospaced` fonts.

2 Equations

Let us see how easy it is to write equations.

$$\Delta = \sum_{i=1}^{N} w_i (x_i - \bar{x})^2. \tag{1}$$

It is a good idea to number equations, but we can have a equation without a number by writing

$$P(x) = \frac{x-a}{b-a},$$

and

$$g = \frac{1}{2}\sqrt{2\pi}.$$

We can give an equation a label so that we can refer to it later.

$$E = -J \sum_{i=1}^{N} s_i s_{i+1}, \tag{2}$$

1

Introduction to LaTeX

©2006 by Harvey Gould
December 5, 2006

1 Introduction

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and

$$g=\frac{1}{2}\sqrt{2\pi}$$

We can give an equation a label so that we can refer to it later.

$$E=-J\sum_{i=1}^{N} s_i s_{i+1}, \tag{2}$$

When you use Word you have to...

- Worry about the look of your final work.

- Manualy set every spacing.
- Remember to center quations.
- Manualy number equations and references (what if you decide to insert another one in the begining).
- Manualy number figures, tables etc.
- Use “special character” dialog to insert many mathematical symbols.
- Pay for it.

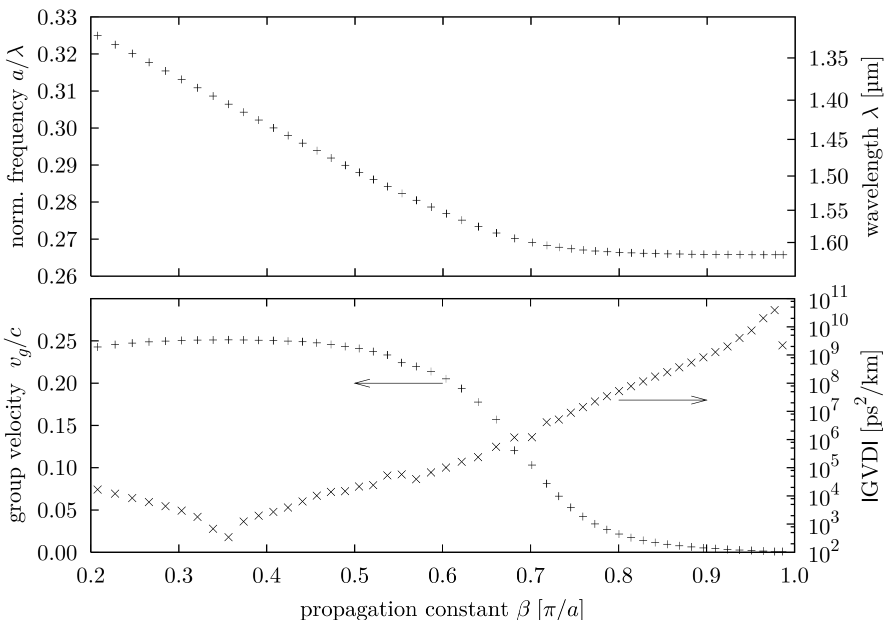
Of course you can use Word if you insist. But you will need to to much more work to have a good result. And you will never have a very good result.

Excel

- Excel is a good software, but...
- It costs money.
- It is designed for financial analysis in office and not for scientific data processing.
- It is very difficult to make a professionally looking plot of scientific data!

Excel: plots

- Good luck making such a plot in Excel:



7 A word about oral presentations

Key rules in oral presentations

Structuring a talk is a difficult task and the following structure may not be suitable. Here are some rules that apply for this solution:

- Exactly two or three sections (other than the summary).
- At **most** three subsections per section.
- Talk about 1min per frame. So there should be about 10 frames, all told.

- A conference audience is likely to know very little of what you are going to talk about. So **simplify**!
- In a 10min talk, getting the main ideas across is hard enough. Leave out details, even if it means being less precise than you think necessary.
- If you show bibliography (required e.g. if you use someone's image) do it at the bottom of the same slide.

Part III
Measurements and Devices

Measurements and Devices: Outline

Contents

General measurement rules

- Estimate the in advance in what range is the measuted quantity.
- Choose proper equipment for your task.
- Never work out of allowed range! You can damage the apparatus or at leas have unreliable results.
- Try to work in the middle of the scale.

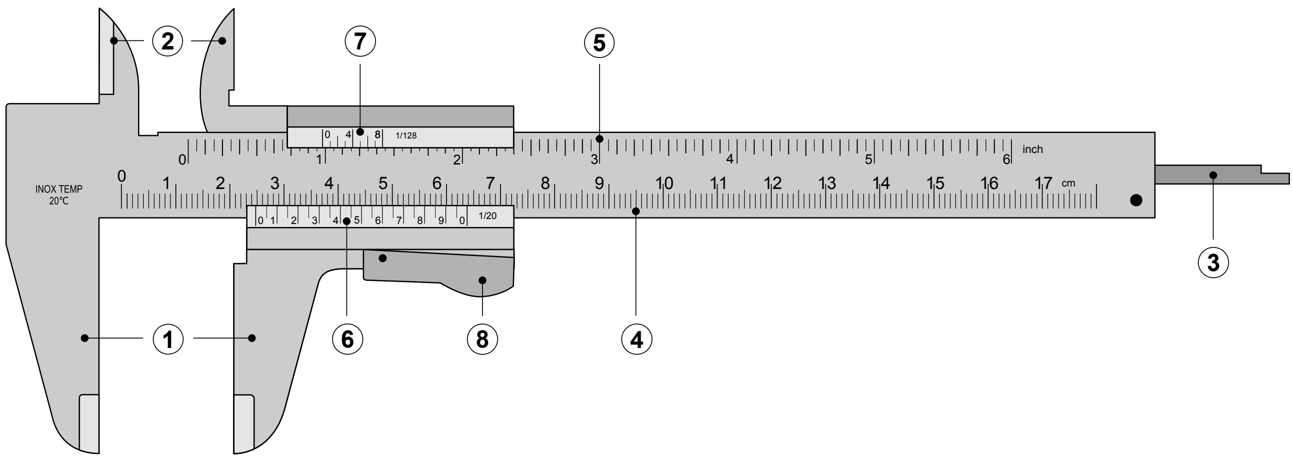
8 Measurements of Basic Quantities

8.1 Length and time

Devices for measuring length

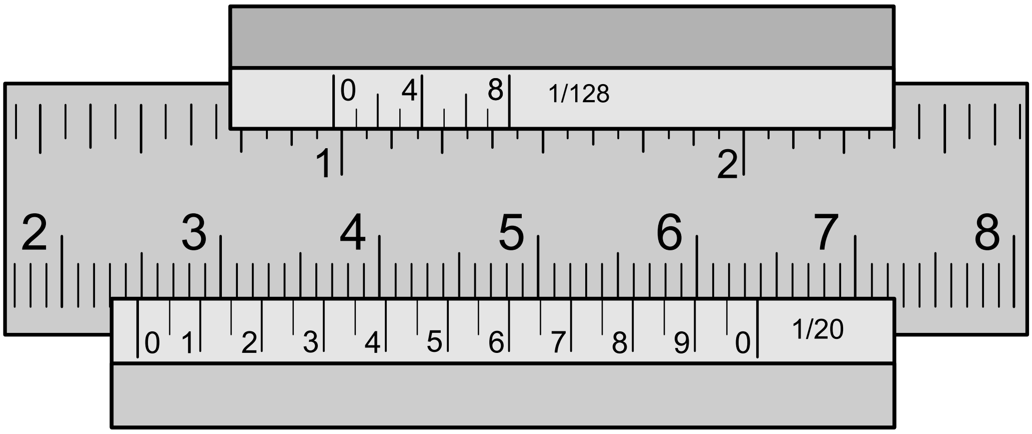
Device	Operating range [m]	Precission [m]
ruler	$0.02 \dots 5.00$	0.5×10^{-3}
caliper	$(1.0 \dots 50.0) \times 10^{-3}$	0.05×10^{-3}
micrometer	$(0.01 \dots 10.00) \times 10^{-3}$	1×10^{-6}
dial indicator	$(0.01 \dots 10.00) \times 10^{-3}$	1×10^{-6}
microscope scale	$(1 \dots 5000) \times 10^{-6}$	0.5×10^{-6}

Caliper

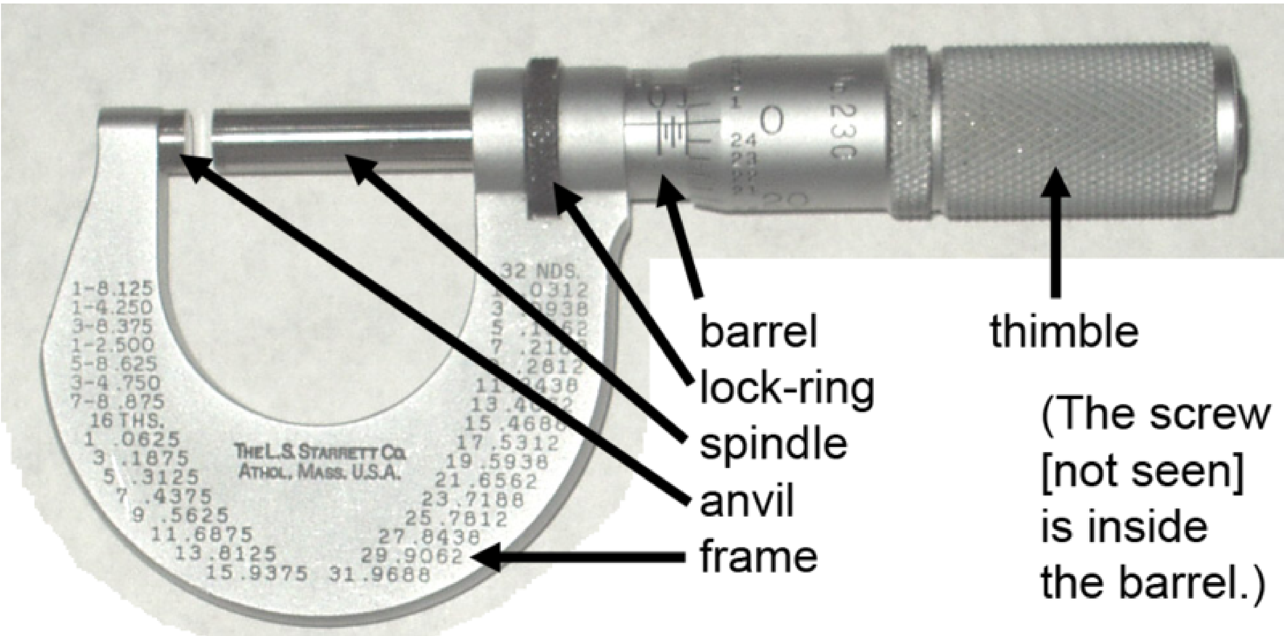


(1) outside jaws, (2) inside jaws, (3) depth probe, (4–5) main scale, (6–7) vernier, (8) retainer.

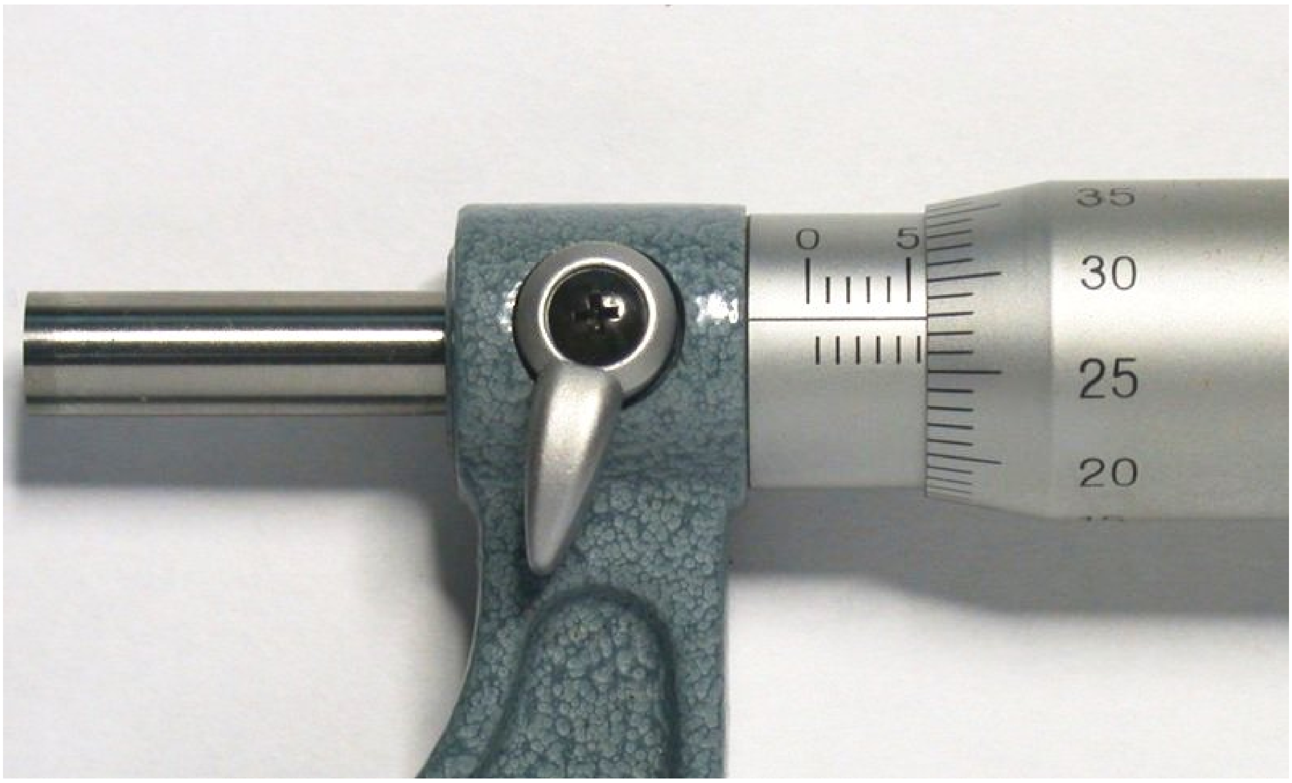
Reading caliper



Micrometer



Reading micrometer



Dial indicator

Measurements of time

- Stopwatches
- Nowadays almost only electronic stopwatches are in use, rarely analog
- Precission of electronic stopwatch is 0.01 s
- Precision of a stopwatch limited by the personal abilities: in practice no better than 0.2 s.
- When measuring periodic phenomenon, measure the time of N (10–20) periods. This will increase precision N times:

$$\Delta T = \Delta t / N$$

8.2 Mass

Mass versus weigth

Definitions 11. **Mass** is the tendency of an object to remain at constant velocity unless acted upon by an external force.
Weight is the force created when a mass is acted upon by a gravitational field.

Weight can be influenced by:

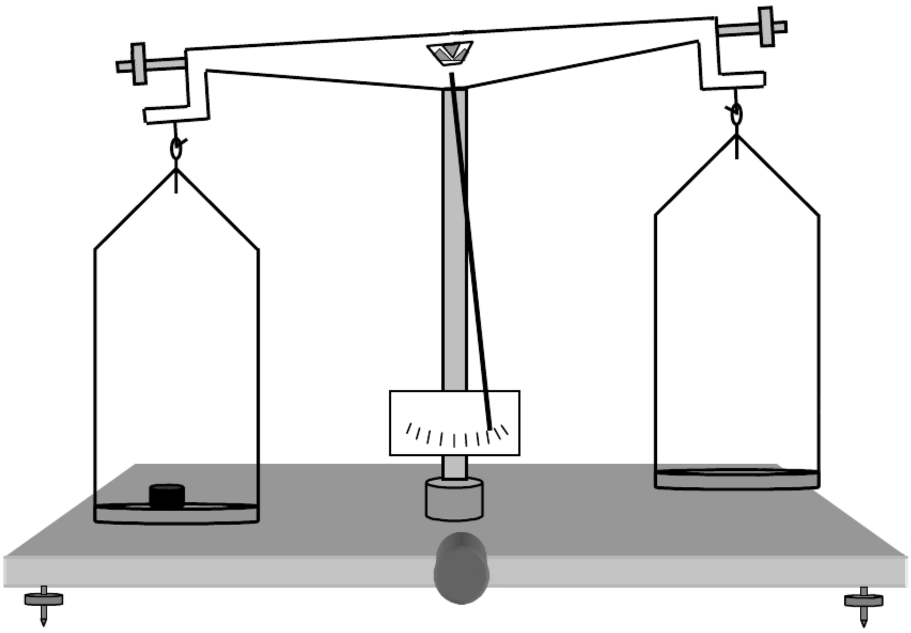
- the geographic place of performing measurement
- buoyancy:
  For a mass at 20 °C, **conventional mass** is the mass of a reference standard of density 8000 $\mathsf{kg/m^3}$ which it balances in air with a density of 1.2 $\mathsf{kg/m^3}$. Devices which measure/compare weights, in fact measure conventional mass. This effect is very small (≈150 ppm).

Types of scales

Device	Operating range [g]	Precission [g]	Typical usage
Spring scale	above 100	50	very big objects
Balance scale	$1 \dots 400$	0.01	big objects, draft mneasurement
Electronic scale	$1 \dots 2000$	0.1	big objects, draft mneasurement
Analytical balance	$1 \times 10^{-3} \dots 200$	5×10^{-5}	precise weighting of small objects
Torsion scale	$1 \times 10^{-2} \dots 5$	5×10^{-4}	precise weighting of small objects

Balance

Balance



Using balance

1. Ensure the balance is horizontal. Balance the scale.
2. Lock the balance.
3. Put the object on one scale.
4. Put the weights on the other scale of total mass close to the one of the object. **Never touch weights by hand. Use scisors for this!**
5. Unlock the balance, observe the pointer.
6. Adjust the weigths by adding them or replacing the smallest one with smaller. Repeat until the smallest wight is used.
7. Read the total weight.

Correcting for buoyancy

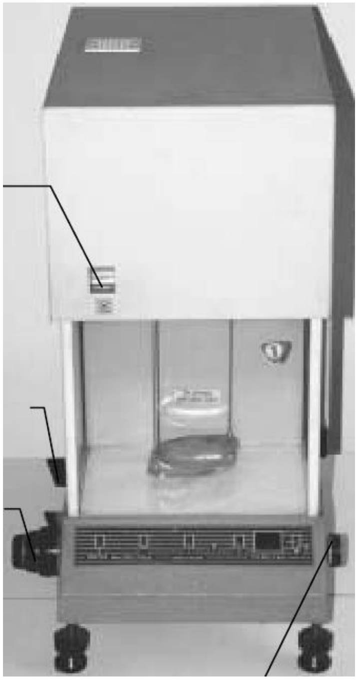
- Balances measure conventional mass.
- To get the real mass use correction
$$m = k\, m_{\mathsf{measured}}$$
where
$$k = 1 + \rho_a \left(\frac{1}{\rho_s} - \frac{1}{\rho_0} \right)$$
ρ_a is the density of air, ρ_s the density of the measured object, ρ_0 the density of the weights.

Electronic scale

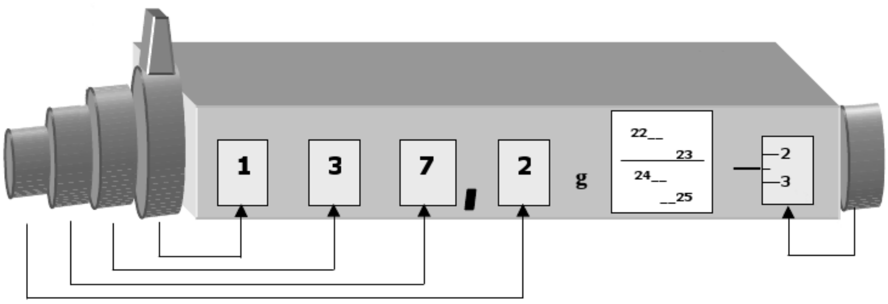
1. Callibrate the scale (see manual).
2. Put the object and read weight.

Analytical scale (electronic)

Analytical scale



Using analytical scale



1. **Always** make draft measurements. Set the scale to the approximate mass of weighted object.
2. **Never** adjust the dials, not put/remove object when the scale is unlocked.
3. **Do not overweight!**

8.3 Temperature

Measuring and controling temperature

- Measuring temperature
  - liquid thermometer
  - thermocouple
  - resitance thermometer
- Controling temperature
  - thermostat

Liquid thermometer

- Uses thermal expansion of liquids.
- Commonly uses alcohol (161K–350K) or mercury (233K–630K).

- Slow.
- Paralactic error.
- In laboratory thermometers (unlike medical ones) the mercury can lower easily. So **dont shake them!**

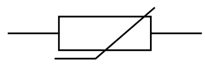
Thermocouple

- Uses Seebeck phenomenon: generation of electromotoric force is circuits made of two metals of different work functions (the energies needed to remove the electron from the metal).
- One end is in the measurement point and the other in the reference temperature.
- Sometimes the reference is realized electrically
- Series of thermocouples can be connected into thermopiles to increase the electromotoric force.
- The electromotoric force is approximately linear function of the temperature difference.
- Suitable for measuring temperatures within big range
- Typical thermocouples: chromel–alumel (type K), chromel-constantan (E), iron-constantan (J), nicrosil-nisil (N), platinium alloys (B, R and S), copper-constantan (T), nikel alloys of molybdenum and cobalt (M).

Resistance thermometer

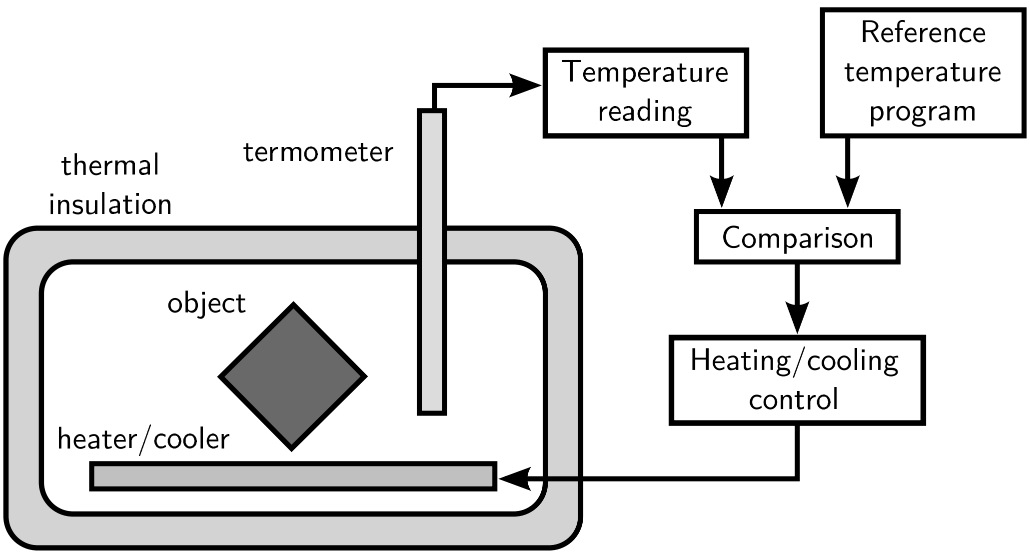
- Uses dependence of resitance on temperature

Definition 12. A **thermistor** is a type of resistor with resistance varying according to its temperature. The word is a combination of *thermal* and *resistor*.



- Not suitable for large range of temperatures (lineach characteristics only in small range)
- More precise than thermocouples
- Slower than thermocouples

Regulation of the temperature



Liquid thermostat

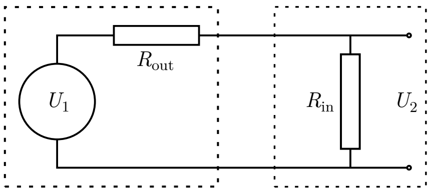
- Consists of a water tank, a pump, a heater and a thermometer
- Desired temperature is chosen by setting the position of the wire
- The heater is turned on and the temperature of the water increases as well as the level of the mercury
- When the desired temperature is reached, the mercury touches the wire and an elecric circuit is closed, which causes the heater to turn of (until the circuit is open again)

9 Electrical Equipment

9.1 Electric Circuits

Input/output resistance

- Each sourcve has some internal resistance: **input resitance** connected in row
- Each connected device has some small current: **output resitance** connected in parallel



$$\frac{U_1}{R_{\mathsf{out}} + R_{\mathsf{in}}} = \frac{U_2}{R_{\mathsf{in}}} = I \quad U_2 = U_1 \frac{R_{\mathsf{in}}}{R_{\mathsf{out}} + R_{\mathsf{in}}}$$

- For good transmission of signal ($U_1 = U_2$) we want $R_{\mathsf{in}} \gg R_{\mathsf{out}}$
- Maximum power transfer is for $R_{\mathsf{in}} = R_{\mathsf{out}}$

DC sources

- Power supplies
- Non-rechargeable batteries
  - zinc-carbon (old, rather not suitable for electronic equipment)
  - alkaine
  - lithium (expensive)
- Rechargeable batteries
  - nickel-cadmium
  - nickel-metal hydride
  - lithium-ion

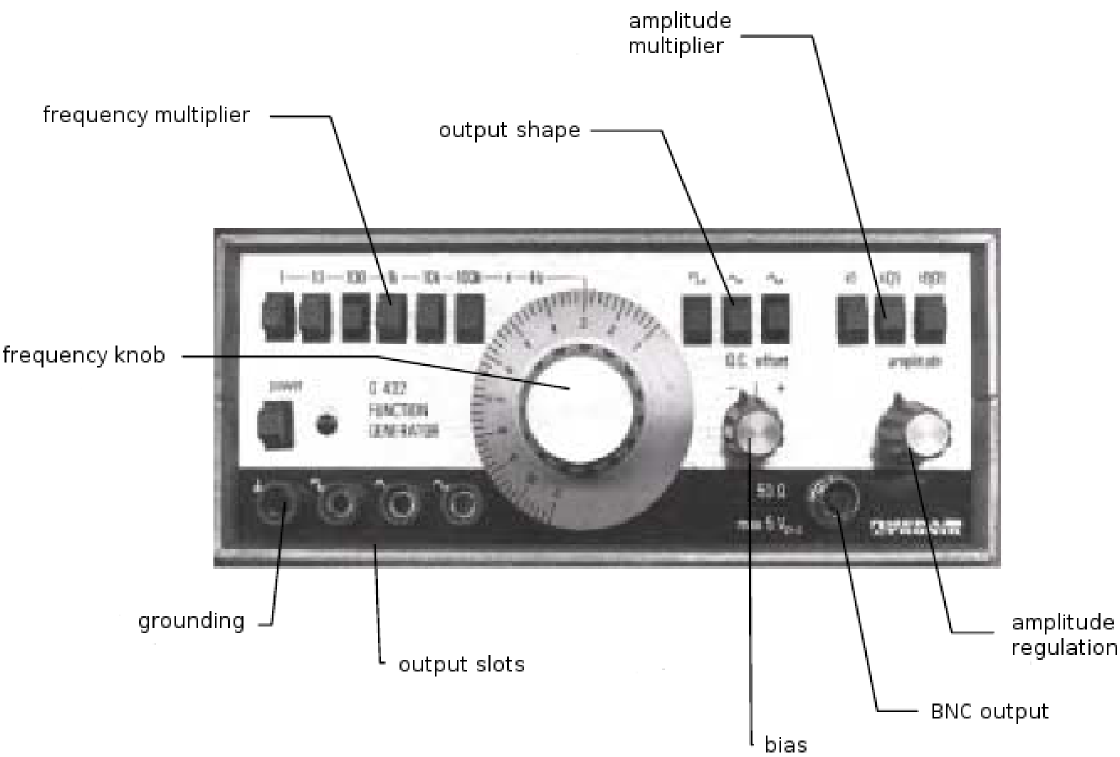
Rechargeable batteries

- nickel-cadmium
  - can suffer “memory effect”: a sudden drop in voltage near initial capacity if charger when not fully-uncharged
  - resitant to low temperature
  - can be safely fully discharged
- nickel-metal hydride

- don't work well in low temperatures: stop working ≈ −20°C, cannot be charged below 0°C
- should not be fully discharged; very small memory effect

- lithium-ion
  - can **never** be fully discharged; no memory effect
  - small drop in voltage in low temperatures; cannot be charged below 0°C
  - when not used, should be kept charged around 40% in low temperature

Periodic AC generators

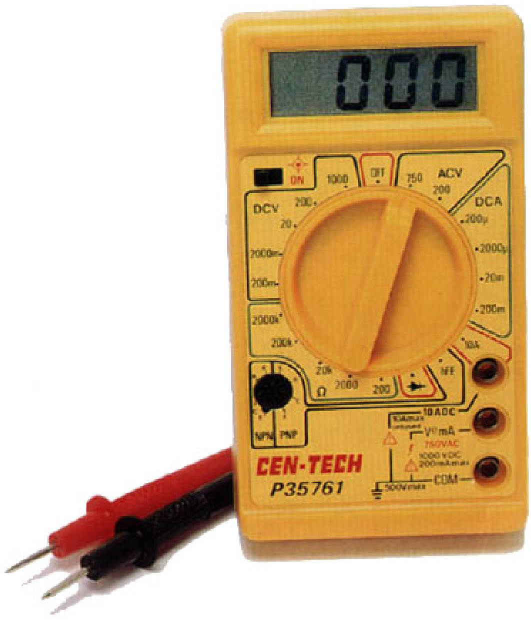


Resistors

color	digits [1-3]	multiplier [4]	tolerance [5]	TC [6]
black	0	10^0		
brown	1	10^1	1%	100ppm
red	2	10^2	2%	50ppm
orange	3	10^3		15ppm
yellow	4	10^4		25ppm
green	5	10^5	0.5%	
blue	6	10^6	0.25%	
violet	7	10^7		
gray	8			
white	9			
gold		10^{-1}	5%	
silver		10^{-2}	10%	
none			20%	

9.2 Analog and Digital Meters

Digital multimeters



- Set larger range than expected measurement first
- Mind the polarisation
- Turn of after use to save battery

Precission of digital multimeters

Display resolution r is the magnitute of the last displayed digit. The error of a digital meter (e. g. voltometer) is larger and can ce computed from

$$\Delta U = pU + eU_R \qquad \frac{\Delta U}{U} = p + U_R \frac{U_R}{U}$$

here p is the basic precission given by the manufacture as the percentage of measured quantity U, e is the range precision and depends on the chosen range U_R.

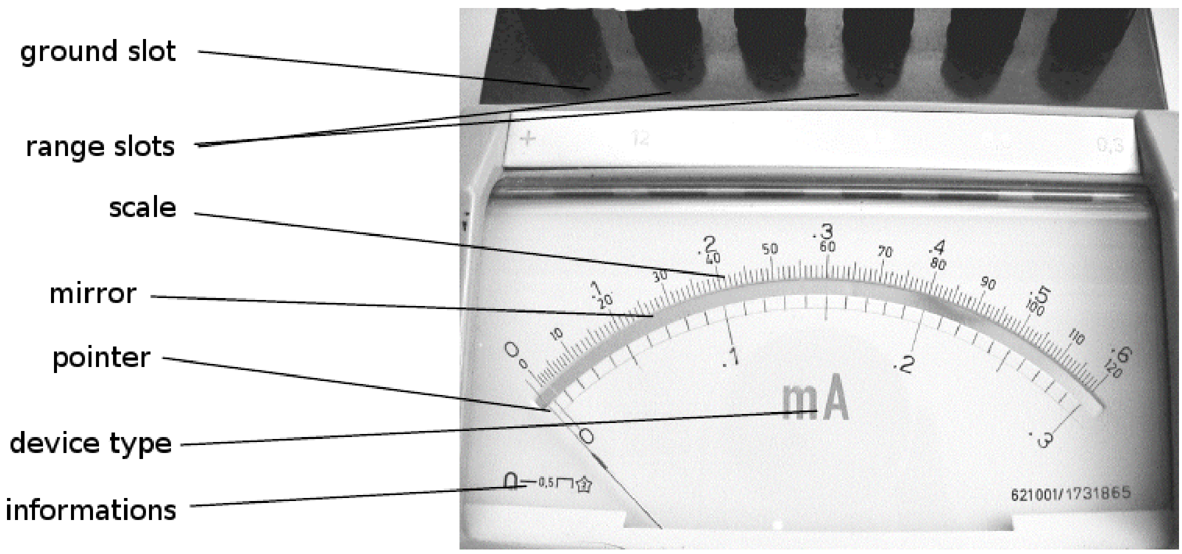
Very often only p is given. In such case one can assume eU_R to be equal to r.

For example if we read $U = 76.43\,\mathrm{mV}$ then $r = 0.01\,\mathrm{mV}$. For $p = 0.05\%$ and $e = 0.01\%$ we have

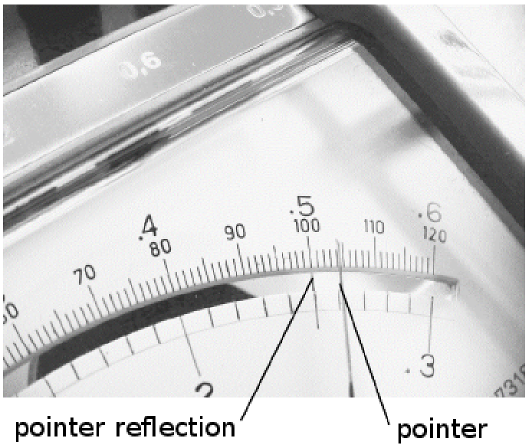
$$\Delta U = 0.05 \times 10^{-2} \times 76.43\,\mathrm{mV} + 0.01 \times 10^{-2} \times 100\,\mathrm{mV} = 0.05\,\mathrm{mV}$$

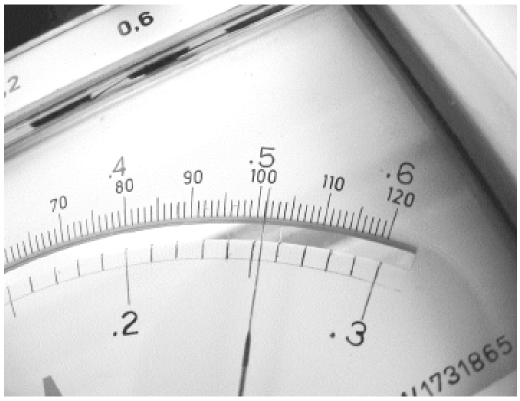
which is 5 times larger than r.

Analog meters

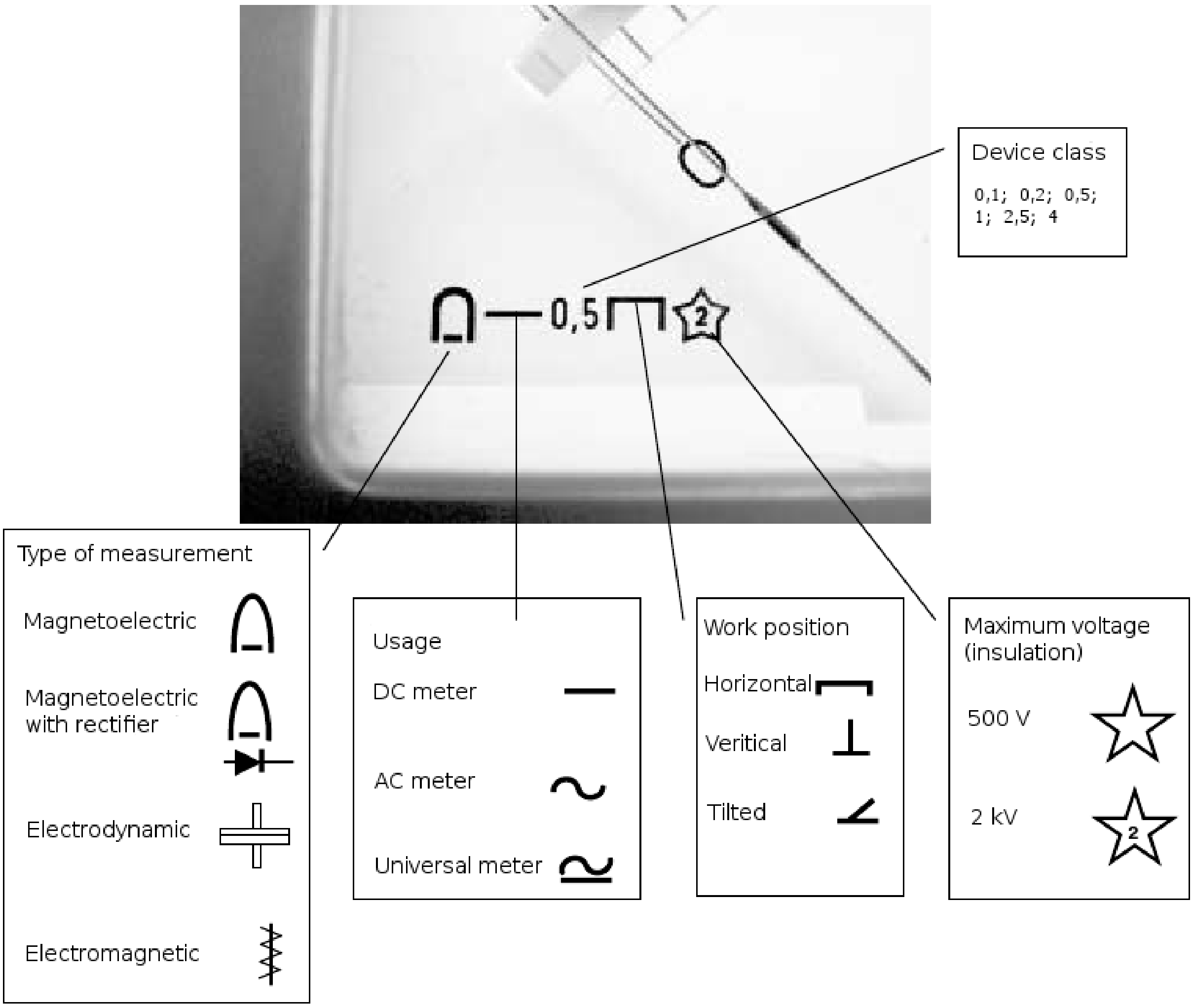


Paralax effect





Analog meters informations



Device class

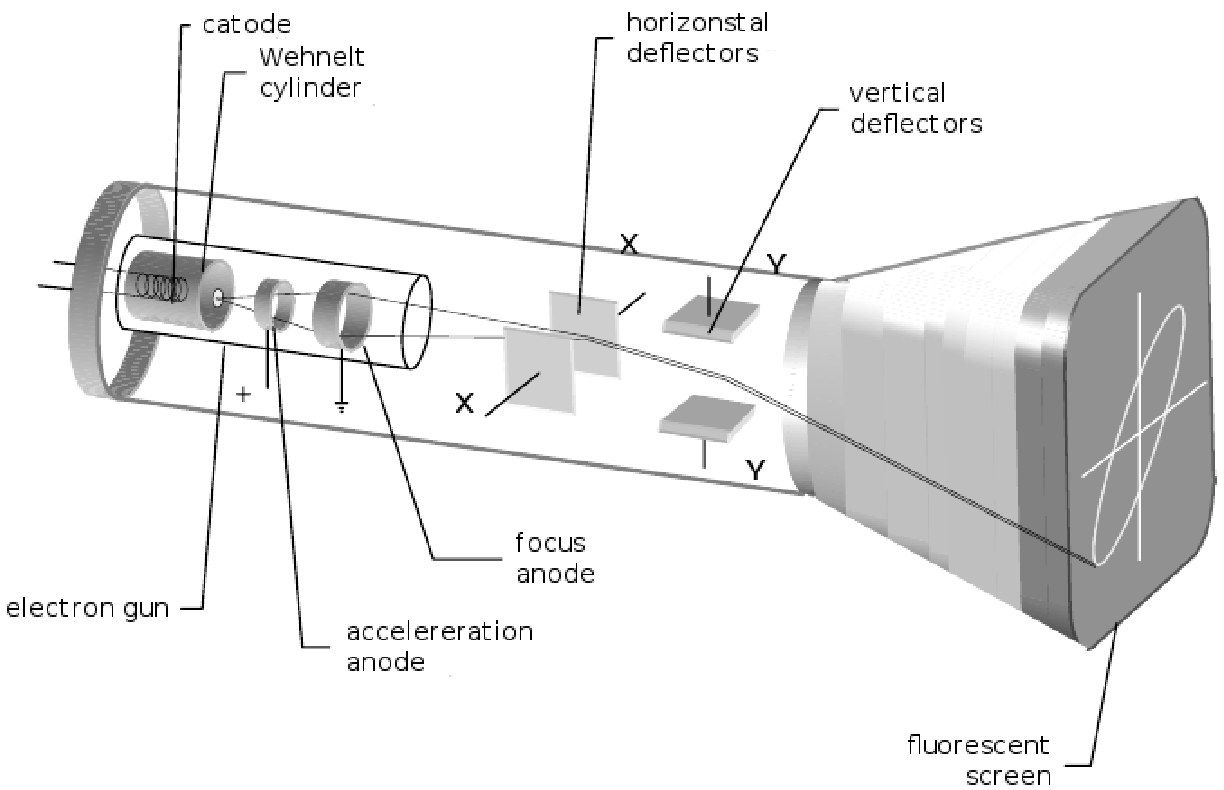
Error of single measurement is

$$\Delta x = \frac{c}{100} R$$

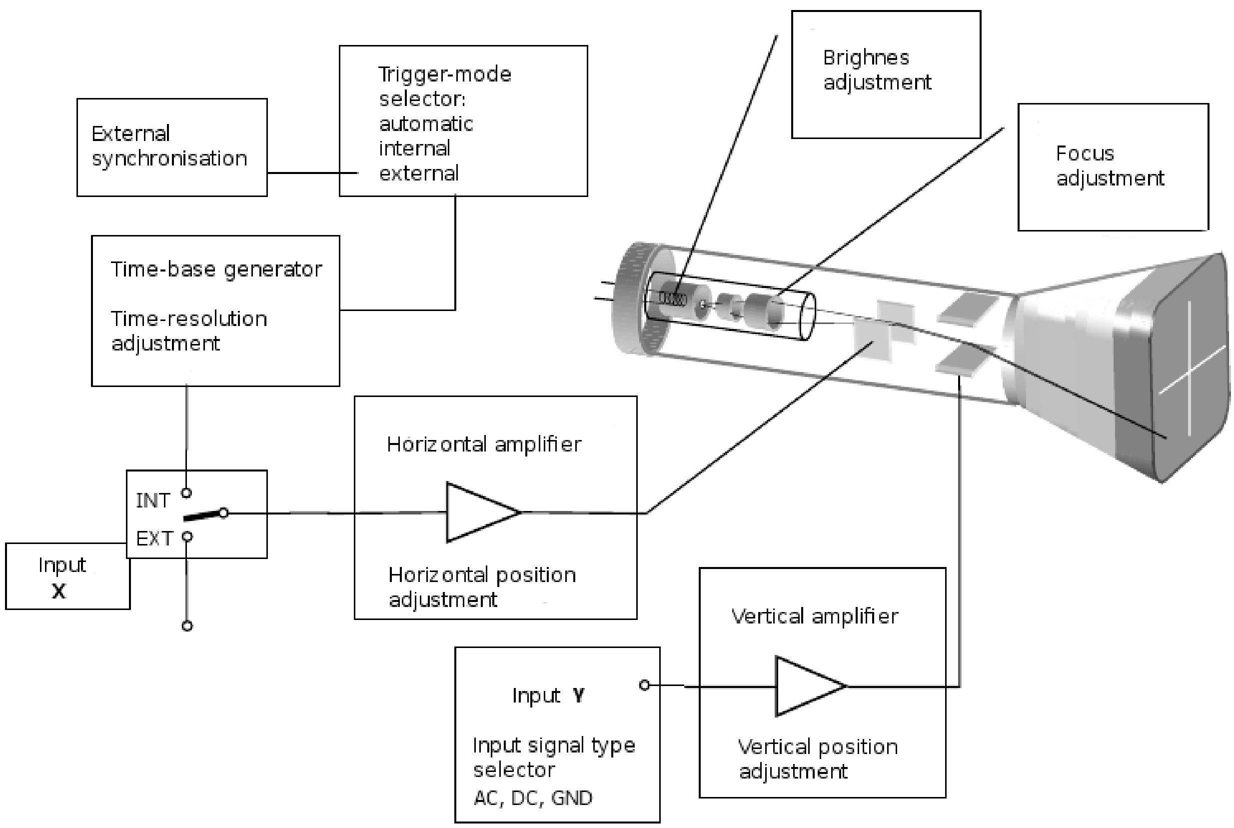
where R is range and c the device class.

9.3 Oscilloscope

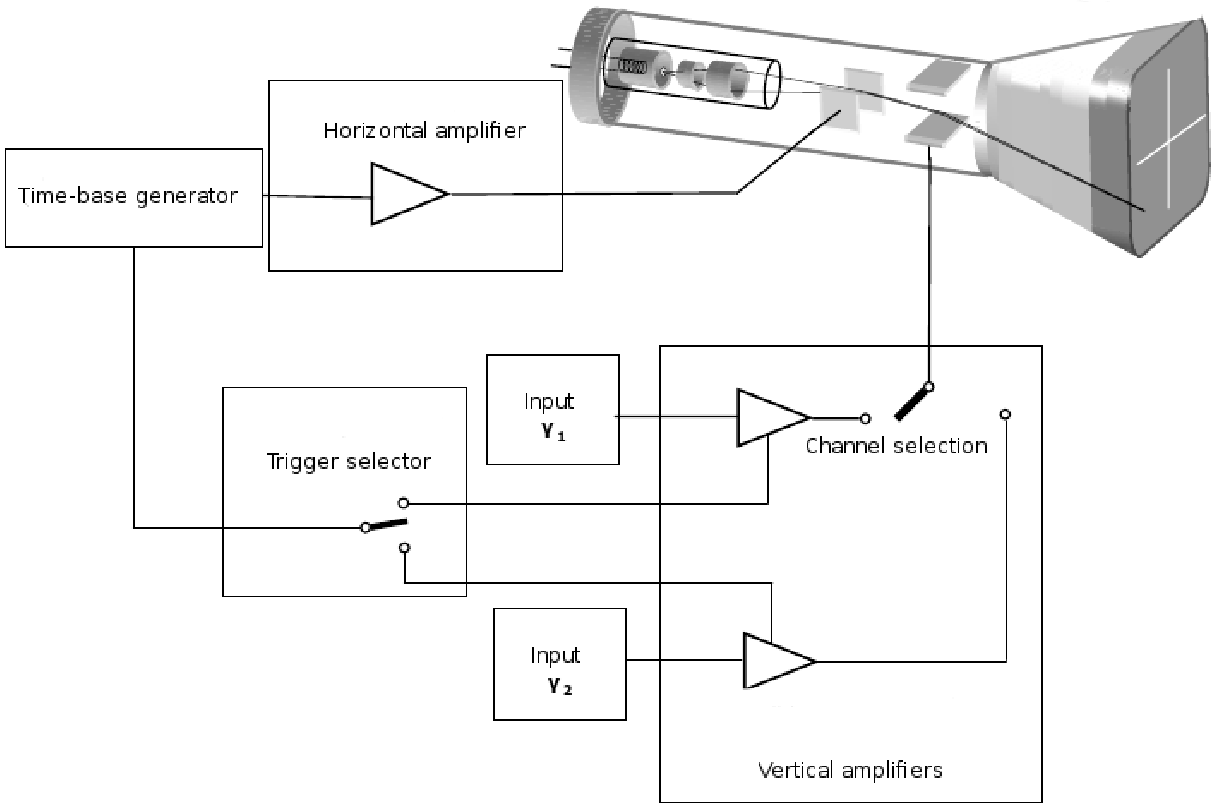
Construction of oscilloscope



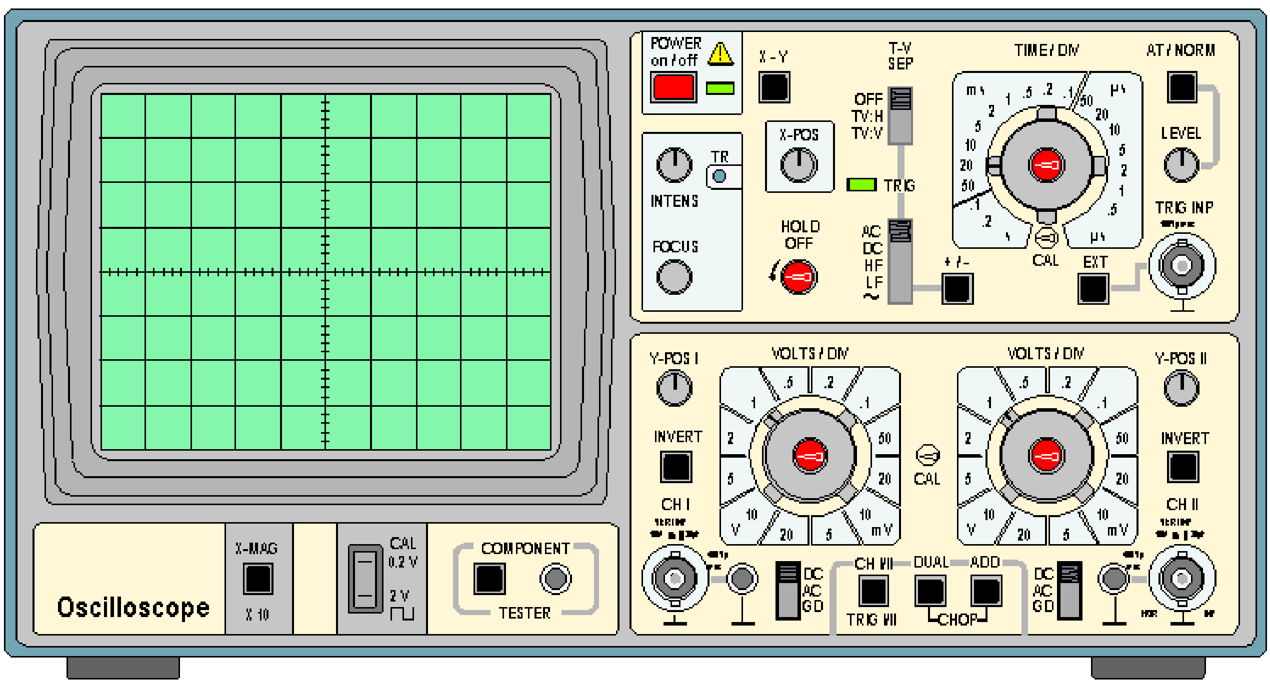
Signal processing in single-channel analog oscilloscope



Signal processing in double-channel analog oscilloscope



Oscilloscope front-panel



Performing measurements with oscilloscope

- Mind that fine-tuning knobs are set in CAL positions.
- Set the sensitivity C: V/div for voltage and ms/div or µs/div for time. For reading the voltage from two channels on both axes set the trace knob to x-y or x-gain to external (EXT).

- Read the amplitude/period of the signal on screen N and translate this to the voltage or time.

$$U = CN$$

- In order to stabilize the image adjust *triggering*. This will start the horizontal scan with constat velocity at raising/dropping edge of the signal.

- The error is

$$\frac{\Delta U}{U} = d_C + \frac{\Delta N}{N}$$

  where d_C is the error of sensitivity given by the manufacturer.